Mathematics 401 (and 701): Introduction to Abstract Algebra

Fall 2023	Tu,Th 3:05–4:20 pm	Physics building 259

Professor: Lenny Ng
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This syllabus last updated August 28.

Course web site: Assignments and other information (likely including course notes) will be posted to Sakai, `https://sakai.duke.edu/`. There is also a rudimentary public course web page at `https://math.duke.edu/~ng/math401f23/` where you can find this syllabus.

Textbook: The required text for this course is *Abstract Algebra: A Geometric Approach* by Theodore Shifrin. I strongly recommend that you read the text concurrently with lectures as we go along.

Office hours: TBA and by appointment (set up in person or by email). If you want to set up an appointment via email outside of scheduled office hours, please keep in mind that I can't usually answer email immediately; on occasion it may take a day for me to respond.

Course TA: Fernando Villegas Negrete (f.vill@duke.edu) is the graduate TA for this course. He is an excellent resource for any algebra-related questions that you may have. He will also hold office hours for this class and will occasionally fill in for me in teaching the class when I need to be out of town.

Course synopsis: The integers and modular arithmetic; polynomials, their roots, and field extensions; groups and symmetry. Applications may include the symmetries of plane tessellations, the impossibility of trisecting an angle with compass and straightedge, the symmetries of platonic solids, modern cryptography, and/or a brief introduction to Galois theory. A side goal of the course is to learn to write clear and correct mathematical proofs.

Prerequisites: Math 221 (linear algebra). If you haven't yet taken Math 221, then please consult with me as Math 401 may not be appropriate for you.

Alternate course: A more advanced version of our course is Math 501, which can be extended to a yearlong course by adding Math 502. However, you can't take Math 501 if you've already taken Math 401, so please plan carefully.

Problem sets: There will be weekly homework sets, tentatively due on Thursdays, to be submitted on Gradescope. You are allowed and encouraged to work with fellow students on the homework; if you do collaborate, please indicate the name(s) of your collaborator(s) on your problem set. Each student must *write up* their problem sets on their own.

Assignments: In addition to the problem sets, this course will have two in-class midterm exams and a final exam. In accordance with Duke schedule, the final exam is on December 15, 7–10 pm.

Your grade will be based on a weighted average of your grades in these components: homework 15%, each midterm 25%, final 35%.

Special note for Math 701 students: You will also be required to write a short essay (under 1 page) explaining the relevance or potential relevance of this course to your particular course of study. This will be due at the time of the final exam.

Topics to be covered: Here is a tentative list of topics, time permitting and subject to change.

- Logic, sets, functions, equivalence relations (sections A.1, A.2, A.3)
- Induction, integers, prime numbers, Euclidean algorithm, Fundamental Theorem of Arithmetic, modular arithmetic (sections 1.1, 1.2, 1.3)
- Rings, integral domains, fields, $\mathbb{Z}_m$, $\mathbb{C}$ (sections 1.4 and 2.3)
- Polynomial rings, division algorithm, remainder theorem, root-factor theorem, Euclidean algorithm for polynomials, unique factorization (section 3.1)
- Roots of polynomials, Fundamental Theorem of Algebra, adjoining elements, Rational Root Theorem, Gauss's lemma (sections 3.2 and 3.3)
- Ring homomorphisms, ideals, isomorphisms, homomorphism theorem, splitting fields (sections 4.1 and 4.2)
- Gaussian integers, primes of the form $a^2 + b^2$, primes of the form $4k + 1$ (section 4.3)
- Groups, symmetry groups, group homomorphisms and isomorphisms (sections 6.1 and 6.2)
- Cosets, Lagrange's theorem, classification of small finite groups, normal subgroups, quotient groups, fundamental homomorphism theorem (section 6.3)
- Group actions, orbits, stabilizers, symmetry groups of regular polyhedra (sections 7.1 and 7.2).