Math 411—Notation and Conventions

Fall 2017

- "$\subset$" = "$\subseteq$": $A \subset B$ means that for all $x \in A$, $x \in B$. Thus if $A \subset B$ and $B \subset A$, then $A = B$.
- Complement: $X \setminus A = X - A = \{x \mid x \in X \text{ and } x \notin A\}$. Note that it is not necessary that $A \subset X$.
- Distributive laws and DeMorgan's laws for sets:

$$\begin{aligned} A \cup \left(\bigcap_i B_i \right) &= \bigcap_i (A \cup B_i) \\ A \cap \left(\bigcup_i B_i \right) &= \bigcup_i (A \cap B_i) \\ A \setminus \left(\bigcup_i B_i \right) &= \bigcap_i (A \setminus B_i) \\ A \setminus \left(\bigcap_i B_i \right) &= \bigcup_i (A \setminus B_i). \end{aligned}$$

- For a map $f : X \to Y$, and subsets $A \subset X$ and $B \subset Y$, the image of A is $f(A) = \{f(x) \mid x \in A\} \subset Y$, and the inverse image of B is $f^{-1}(B) = \{x \mid f(x) \in B\} \subset X$. A superscript $^{-1}$ will always mean inverse in the sense of maps, not in the sense of reciprocals.
- $\forall$ = "for all"; $\exists$ = "there exist(s)"; $\hookrightarrow$= "injective (one-to-one) map"; $\twoheadrightarrow$= "surjective (onto) map".
- $\mathbb{Z}$ = integers, $\mathbb{Q}$ = rational numbers, $\mathbb{R}$ = real numbers, $\mathbb{C}$ = complex numbers.
- Intervals in the real line: $(a, b) = \{x \mid a < x < b\}$; $[a, b] = \{x \mid a \le x \le b\}$; $[a, b) = \{x \mid a \le x < b\}$; $(a, b] = \{x \mid a < x \le b\}$.
- Balls in $\mathbb{R}^n$: $B(x_0, r) = \{x \in \mathbb{R}^n \mid \|x - x_0\| < r\}$; $\overline{B}(x_0, r) = \{x \in \mathbb{R}^n \mid \|x - x_0\| \le r\}$.
- "A is a necessary condition for B" = "B $\Rightarrow$ A"; "A is a sufficient condition for B" = "A $\Rightarrow$ B"; "A is a necessary and sufficient condition for B" = "A $\Leftrightarrow$ B".
- TFAE = "the following are equivalent", i.e., each condition is a necessary and sufficient condition for the others.
- A set A is *countable* if there is an injective map $A \hookrightarrow \mathbb{Z}$. $\mathbb{Q}$ is countable while $\mathbb{R}$ and $\mathbb{C}$ are not.