Mathematics 612: Algebraic Topology II

Spring 2022   Mondays, Wednesdays 10:15–11:30 am   Physics 205

Professor: Lenny Ng   Office: Physics 216
E-mail: `ng AT math.duke.edu`   Office hours: TBA

Sakai site: I'll be using Sakai for assignments and other information; please contact me if you need access and don't automatically have it. There is also a public course web page at `https://services.math.duke.edu/~ng/math612s22/`.

Course synopsis: This is the follow-up course to Math 611. Here are the topics that I plan to cover in the course:

- Singular cohomology, cup product, Poincaré duality

- Differential forms, de Rham cohomology, Poincaré duality (again but now via de Rham cohomology), Künneth Theorem

- Čech cohomology, presheaves

- Spectral sequences, double complexes, equivalence of cohomology theories, Leray-Serre spectral sequence

- (to the extent that time permits:) Vector bundles, Thom isomorphism.

Textbooks: We will use two texts in this class:

- *Algebraic Topology* by Allen Hatcher. This is conveniently available for free online at `http://pi.math.cornell.edu/~hatcher/AT/ATpage.html`, though I recommend that you also purchase a physical copy of the book.

- *Differential Forms in Algebraic Topology* by Raoul Bott and Loring Tu. This is the "official" textbook for the course, and in my opinion it's an essential book on every topologist's and geometer's shelf.

We will use Hatcher for the first portion of the course (the first few weeks), when we discuss singular cohomology. The rest of the course will be based on Bott and Tu.

Prerequisites: Math 611 or familiarity with equivalent material (fundamental group, simplicial/singular homology, CW complexes; essentially the first two chapters of Hatcher). Math 620 or familiarity with basic differential topology (smooth manifolds, tangent/cotangent bundle, differential forms) will also be assumed, but this isn't an ironclad prerequisite; please talk to me if you don't have previous background in smooth manifolds.

Office hours: Regular office hours are TBA but will probably be on Mondays and Tuesdays. You can also make appointments to meet me outside of office hours (set up in person or by email).

Course duration: This class will run through the end of *undergraduate* courses, even though graduate courses ostensibly end a week earlier. This means that our last day of class is Wednesday April 20.

Assignments: There will be weekly problem sets, which I expect to typically be due on Wednesdays. Please check Sakai for posted problem sets. You can work with other students in the class on the homeworks, but please write up your problem sets on your own. My current plan is to ask you to submit your problem sets electronically as a PDF (preferentially), or alternatively as a paper hard copy (if you give me a hard copy, I'll probably scan it and then grade the PDF).

Final exam: The final for this course will be a take-home final exam. My plan is to make it available on April 20 (the last day of our class) and have it be due at 12 noon on Monday April 25 (our final exam time slot). Please let me know if you have issues with this timing; I can be flexible as long as we follow university constraints.

Grading: Your grade is based on the problem sets and final exam.

Course notes: A version of my course notes for Math 612, from fall 2014 (!), is available on Sakai or at the web page `https://services.math.duke.edu/~ng/math612s22/`. Please note that they may contain errors, and they aren't a substitute for class attendance.

COVID considerations: We will follow university guidelines and everyone should wear a mask during in-person classes and office hours. It is crucial that you do not come to class if you have COVID symptoms or if you're asked by the university to quarantine. If you do need to miss class due to quarantine or isolation, please let me know as soon as possible and we can make arrangements; since there are lecture notes online, it should be fairly simple to make sure you miss as little as possible.

COVID considerations, part II: For (at least?) the first 3 classes (January 5, 10, and 12), our class will be held online, as per university rules. You can find the Zoom link on Sakai or on the course web page. We'll switch back to in-person classes as soon as we're allowed.