Mathematics 612: Algebraic Topology II

Spring 2025 | Wednesdays, Fridays 1:25–2:40 pm | Physics 259

Professor: Lenny Ng
E-mail: ng AT math.duke.edu
Office: Physics 216
Office hours: TBA, and by appointment

Canvas site: I'll be using Canvas for assignments and other information; please contact me if you need access and don't automatically have it. There is also a public course web page at https://math.duke.edu/~ng/math612s25/.

Course synopsis: This is the follow-up course to Math 611. Here are the topics that I plan to cover in the course:

- Singular cohomology, cup product, Poincaré duality
- Differential forms, de Rham cohomology, Poincaré duality (again but now via de Rham cohomology), Künneth Theorem
- Čech cohomology, presheaves
- Spectral sequences, double complexes, equivalence of cohomology theories, Leray-Serre spectral sequence
- (to the extent that time permits:) Vector bundles, Thom isomorphism.

Textbooks: We will use two texts in this class:

- *Algebraic Topology* by Allen Hatcher. This is conveniently available for free online at http://pi.math.cornell.edu/~hatcher/AT/ATpage.html, though I recommend that you also purchase a physical copy of the book.
- *Differential Forms in Algebraic Topology* by Raoul Bott and Loring Tu. This is the "official" textbook for the course, and in my opinion it's an essential book on every topologist's and geometer's shelf.

We will use Hatcher for the first portion of the course (the first few weeks), when we discuss singular cohomology. The rest of the course will be based on Bott and Tu.

Prerequisites: Math 611 or familiarity with equivalent material (fundamental group, simplicial/singular homology, CW complexes; essentially the first two chapters of Hatcher). Math 620 or familiarity with basic differential topology (smooth manifolds, tangent/cotangent bundle, differential forms) will also be assumed, but this isn't an ironclad prerequisite. If you don't have previous background in smooth manifolds, I strongly recommend that you read chapters 1–3 (and ideally also chapter 110 of *An Introduction to Smooth Manifolds* by John Lee. We won't need any of the material on smooth manifolds until about 1/3 of the way through the course. Please talk to me if you have any questions.

Office hours: TBA. You can also make appointments to meet me outside of office hours (set up in person or by email).

Class meetings: This class will run through the end of *undergraduate* courses, even though graduate courses ostensibly end a week earlier. This means that our last day of class is Wednesday April 23. Also please note that Wednesday January 8 follows a Monday schedule; our first day of class is Friday January 10.

Assignments: There will be weekly problem sets, which I expect to typically be due on Wednesdays on Gradescope. Please check Canvas for posted problem sets. You can work with other students in the class on the homeworks, but please write up your problem sets on your own.

Final exam: The final for this course will be a take-home final exam. My plan is to make it available on April 23 (the last day of our class) and have it be due during our final exam time slot on May 1. Stay tuned for details. Please let me know if you have issues with this timing; I can be flexible as long as we follow university constraints.

Grading: Your grade is based on the problem sets and final exam.

Course notes: A version of my course notes for Math 612, from fall 2014 (!), is available on Canvas or at the web page `https://math.duke.edu/~ng/math612s25/`. Please note that they may contain errors, and they aren't a substitute for class attendance.