Mathematics 633: Complex Analysis

Spring 2021	Tu, Th 8:30–9:45 am ET	Online only

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Public course web site: There is a rudimentary course web site at
`http://www.math.duke.edu/~ng/math633/`.
I'll also use Sakai and probably Gradescope; please stay tuned.

Spring 2021 format: This course will be held **online on Zoom** only. I'll post recordings of the Zoom lectures, as well as lecture notes, on Sakai.

Class meetings on Zoom: We will be meeting each Tuesday and Thursday from **January 21** to **April 20**, except for February 18 and March 9. If you're registered for the course, then you can find the link to our Zoom meetings on Sakai. If you're not registered but want to check out the course, and you're a Duke student, you should be able to join the Sakai site yourself and get the Zoom link that way. Please let me know if you have any issues with this.

Office hours on Zoom: TBA, and by appointment.

Textbook: The required text for this course is *Complex Analysis*, 3rd edition, by Lars Ahlfors. I am aware that the book is fairly pricey but if cost is an issue, please note that used copies are pretty readily available. Another good reference is *Functions of One Complex Variable* by John B. Conway; my approach to the course is informed by some mix of Ahlfors and Conway.

Prerequisites: Officially, familiarity with real analysis at the level of Math 532 (Math 631 is even better but not required). More explicitly, you need to be comfortable with the following:

- point set topology at the level of Ahlfors Chapter 3, section 1; it's also helpful if you're familiar with path homotopy and simple connectivity
- the Bolzano–Weierstrass Theorem, the Heine–Borel Theorem, the Lebesgue covering lemma
- uniform continuity, absolute and uniform convergence, the Weierstrass M-test
- the Arzela–Ascoli Theorem.

Alternate course: For undergraduates, we also offer Math 333 as an alternate course in complex analysis. Compared to Math 333, our course will be faster-paced and at a conceptually higher level. Typically students don't take both Math 333 and Math 633.

Homework: There will be weekly homework sets, tentatively due on Thursdays and to be submitted via Gradescope. I will make homeworks available on Sakai a week before they're due. In general, late homeworks will not be accepted.

Exam: There will be a take-home exam due during finals period. For this exam, you will be allowed to consult your notes, the homework, and the book, but no other sources.

Grading: Your grade will be based on $2/3$ homework, $1/3$ final exam.

Topics to be covered: This course will follow the official syllabus for Math 633 as it's been adopted for the qualifying requirement for the Duke math graduate program. Here's that syllabus (without the optional extra topics, which I don't think I'll cover):

- Holomorphic (aka complex analytic) functions, including: definition; branch of log; Cauchy–Riemann equations; as conformal maps; Möbius transformations.
- Integration, including: power series representations; Cauchy's Estimate; zeros of analytic functions; Liouville's Theorem; Maximum Modulus Theorem; Cauchy's Theorem and Integral Formula; counting zeros; Open Mapping Theorem.
- Singularities, including: classification; Schwarz's Lemma and classification of conformal automorphisms of sphere; Laurent Series; Casorati–Weierstrass Theorem; residues; Argument Principle; Rouché's Theorem.
- Space of holomorphic functions, including: Hurwitz's Theorem; Montel's Theorem; Riemann Mapping Theorem; infinite products, the gamma function.
- Analytic continuation, including: germs and analytic continuation; Monodromy Theorem; Riemann surfaces (examples).