

Minicourse: Morse Theory and Floer Homology

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Mondays, Wednesdays 1:25–2:40pm (please note schedule change!)

Fall 2025: begins Monday September 29; ends Wednesday October 29

Morse theory has historically served as an important intermediary between differential geometry and algebraic topology, and it has gained further importance in topology in the last few decades through the work of Floer, Witten, and many others. This minicourse will provide a rapid introduction to Morse theory with the goal of defining Morse homology. We'll then pivot to Floer homology, which can be thought of as an infinite-dimensional analogue of Morse homology, and discuss what's different in this setting.

Topics I plan to discuss:

- Morse theory: Morse functions, surgery and handles, Morse inequalities, Morse–Smale pairs and gradient-like flows
- Morse homology: the Morse–Smale–Witten complex, orientations and signs, Poincaré duality
- Floer homology: an overview of selected flavors, probably instanton Floer homology and symplectic Floer homology. (This will probably be very rapid because of time constraints.)

Everyone is welcome and I'll try to keep prerequisites to a minimum. It will be helpful, but not completely necessary, to have a basic familiarity with algebraic topology along the lines of Math 611, and smooth manifolds along the lines of Math 620.

I have some course notes from a previous version of this minicourse:

math.duke.edu/~ng/math790s15/Morse.pdf

If you'd like further reading material, there are now many good books on Morse theory and Morse homology, some with discussions of Floer theory as well. These include:

- Liviu Nicolaescu, *An Invitation to Morse Theory*
- Michèle Audin and Mihai Damian, *Morse Theory and Floer Homology*
- Augustin Banyaga and David Hurtubise, *Lectures on Morse Homology*
- John Milnor, *Morse Theory* (a classic, but doesn't cover recent developments).

The first two have the added virtue of being electronically available through the Duke Libraries web site.