Math 641, Spring 2023

Course syllabus
(Updates in blue. Last updated: Feb 13)

Instructor: Nicholas Cook

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Office hours: TuTh 9:30–10:30, We 1–2, or by appointment, in Physics 029A.

Course description. Martingales, Markov chains from an advanced viewpoint, ergodic theory, Brownian motion and its applications to random walks, Donsker's theorem and applications; multidimensional Brownian motion and connection to PDE (time permitting); basic large deviations theory (time permitting). Prerequisite: Statistical Science 711 or Mathematics 631. (Course requires a knowledge of measure theory.)

Text: R. Durrett. Probability Theory and Examples (5th edition),
We will proceed through Chapters 4–9 roughly in order (skipping some sections depending on time limitations). The text will be made available on the class Sakai page. If we have time for basic large deviations theory then notes will be posted (the standard reference is Dembo & Zeitouni's *Large deviations Techniques and Applications*).
Additional references:

- P. Billingsly. *Probability and Measure*
- D. Williams. *Probability with Martingales* (Part B)
- O. Kallenberg. *Foundations of Modern Probability Theory*

For review of prerequisite probability material see for instance: Durrett Chapters 1–3, Williams Part A, notes of Greg Lawler (on Sakai), and Section 1.1 of T. Tao's *Topics in Random Matrix Theory* (available on his webpage).

Class meetings. Classes will be held Mondays and Wednesdays at 10:15–11:30am in Gross Hall room 324. The first lecture will be on Wednesday Jan 11th and the last lecture on Wednesday April 26th.

Outside of class. Due to time constraints, some material may be skipped in lectures and assigned as outside reading (e.g. some details of proofs, important examples).
There will be a course Sakai page where homework, handouts and announcements will be posted. Homework will be submitted via Gradescope.
Emails: You can expect a response from me within 24 hours during regular working hours M–F. (So if you email me Friday afternoon you may not get a response until the following Monday.)

Homework. There will be 7 biweekly homeworks due on Fridays at 11:59pm EST, to be submitted via Gradescope, with the first homework due on Jan 20th. In Gradescope you must tag the pages of your uploaded assignment with problem numbers. You should make sure your uploaded writeup is legible. You are free to collaborate with other students on the homework, but you must write up your own solutions, *in your own words*, and acknowledge your collaborators at the top (I emphasize: no penalty whatsoever for collaborating!). Late homework will not be accepted.

Exams. There will be one in-class midterm on Wednesday March 22nd, based on the material of homeworks 1–4 (roughly the first 14 lectures – a precise range will be announced closer to the date of the midterm). The final exam will be held Saturday May 6th 9am–noon in Gross Hall room 324 (the same place class is held). The final exam will be cumulative. There will be no make-up exams, except as required by the University Policy. You may not discuss exams until I authorize you to do so (be aware that in some circumstances a student may need to take the exam at a later time).

Qualification for math PhD program. This is a qualifying eligible course for the math PhD program. To qualify in this course, students must receive at least an A- on the final exam. Students may not take the exam without being enrolled in the course.

Evaluation. Your grade will be a weighted average of your homework, midterm and final grades, with the following weights: Homework 40%, Midterm 20%, Final 40%. Each homework will be graded out of 100%. Your lowest homework score will be dropped, and the remaining homeworks will carry equal weight in your final homework average.

Academic integrity. As members of the Duke community, students are expected to uphold the Duke Community Standard.

Safety. We will adhere to Duke's COVID safety protocols. As of this writing, masks are not required in classrooms. When not required they are still encouraged.

Topics to be covered:

(1) Martingales (≈ 8 lectures)
- Review of conditional expectation
- Almost-sure convergence
- L^p convergence, Doob's inequality
- Uniform integrability, L^1 convergence, Lévy's 0–1 law
- Optional stopping theorems
- Backward martingales
- Important examples and applications: random walks, branching processes, Polya urns

(2) Markov chains (≈ 7 lectures)
- Transition probabilities, construction, Markov property
- Strong markov property, recurrence and transience
- Convergence to equilibrium, coupling
- Applications to random walks

(3) Ergodic theory (≈ 3 lectures)
- Stationary sequences, invariant measures
- Birkhoff ergodic theorem
- Subadditive ergodic theorem (only statement and a few applications)

(4) Brownian motion (≈ 8 lectures)
- Construction of Wiener measure
- Markov property, Blumenthal 0-1 law
- Strong Markov property, reflection principle
- Sample path properties, zero set
- Donsker's theorem and applications

(5) Additional topics (time permitting)
- Itô's formula
- Multidimensional Brownian motion and connection to PDE
- Basic large deviations theory