Math 641, Spring 2021

Course syllabus

(Updates in blue. Last updated: Jan 26)

Instructor: Nicholas Cook

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Office hours: Mo 1–2pm, Th 1:30–2:30pm, or by appointment.

Course description.

Official course catalogue description: Designed to be a sequel to Statistical Science 711. The basic topics are: martingales, Markov chains from an advanced viewpoint, ergodic theory, Brownian motion and its applications to random walks, Donsker's theorem and the law of the iterated logarithm, and multidimensional Brownian motion, connection to PDE's. For those who have not had 711, we will prove the law of large numbers using martingales and obtain versions of the central limit theorem from Donsker's theorem. Course requires a knowledge of meaure theory. Prerequisite: Statistical Science 711 or Mathematics 631.

Texts: We will mainly follow Durrett's book (5th edition), proceeding through Chapters 4–9 roughly in order (skipping some sections depending on time limitations). The text will be made available on the class Sakai page. Amir Dembo's lecture notes (available on his webpage) are also a good reference.

Course structure. This course is designed to be taken remotely and synchronously.

Class meetings. Will be held over Zoom at the scheduled time of noon–1:15pm EST on Tuesdays and Thursdays. You should feel free to speak up and ask questions, but please keep your microphone off otherwise.

If you are in a time zone that makes it difficult to participate, please let me know as early as possible so I can see what I can do to accommodate.

Homework. There will be 7 biweekly homeworks due on Tuesdays at noon EST, to be submitted via Gradescope, with the first homework due on Feb 2nd. In Gradescope you must tag the pages of your uploaded assignment with problem numbers. You should make sure your uploaded writeup is legible. You are free to collaborate with other students on the homework, but you must write up your own solutions, *in your own words*, and acknowledge your collaborators at the top (I emphasize: no penalty whatsoever for collaborating!). Late homework will not be accepted.

Exams. There will be one "take-home" midterm and a "take-home" final exam. You are free to consult outside resources such as other texts or lecture notes, but you may not consult with any person but me (honor code) – this includes posting to forums or message boards. You will be given a common 24 hour window in which to complete the midterm, and a 48 hour window for the final. The midterm will be due Friday of 8th week (March 12th)). The final will be due at noon on April 29th. Late submissions will not be accepted, except as required by the University Policy. You may not post questions about exams until they have been graded and returned to all students (be aware that in some circumstances a student may need to take the exam at a later time).

Office hours etc. My office hours will be held in the same Zoom meeting room as classes. I will try to respond to emails within 24 hours during regular working hours M–F. (So if you email me Friday afternoon you may not get a response until the following Monday.)

Evaluation: Your grade will be a weighted average of your homework, midterm and final grades, with the following weights: Homework 50%, Midterm 20%, Final 30%. Each homework will be graded out of 100%. Your lowest homework score will be dropped, and the remaining homeworks will carry equal weight in your final homework average.

Qualification for math PhD prorgram. This is a qualifying eligible course for the math PhD program. To qualify in this course, students must receive at least an A- on the final exam. Students may not take the exam without being enrolled in the course.

Academic integrity. As members of the Duke community, students are expected to uphold the Duke Community Standard.

Technology requirements and policies: You will need a stable internet connection to join Zoom meetings and submit your homework and exams. Supporting documentation for Gradescope is available here. Technical problems with Gradescope not addressed there must be communicated to me as early as possible. Technical problems with Zoom or Sakai should be directed to the Duke OIT Service Desk.

Students with limited access to computers and internet may request a loaner laptop and/or wifi hotspot here.

Student recording of lectures is not permitted. Unauthorized distribution of recordings is a cause for disciplinary action by the Judicial Board. The full policy on recoding of lectures falls under the Duke University Policy on Intellectual Property Rights.

Course etiquette: (May be expanded/refined as we advance in the course.) To promote an interactive environment you are encouraged to have your cameras on during class meetings, though this is not required. Please be mindful of your surroundings and try to place yourself somewhere that won't have distractive objects or activity in the background. You should feel free to speak up and ask questions, but please keep your microphone off when not speaking.

Support Services. This COVID-19 pandemic is a stressful time for all of us. I urge you to explore the support services available through CAPS and DukeReach.

Topics to be covered.

(1) Martingales: Weeks 1–4 (≈ 7 lectures)
  - Review of conditional expectation; Almost sure convergence, upcrossing inequality, L^p convergence, Doob's inequality, Uniform integrability, L^1 convergence, Levy 0-1 law, Optional stopping theorem, Backward martingales, Important examples and applications: branching processes, Polya urns, etc.

(2) Markov chains: Weeks 5–8 (≈ 7 lectures)
  - Transition probabilities, Rigorous construction -Markov property, strong markov property -Rigorous construction -Recurrence and transience -Convergence to equilibrium, coupling -Important examples

(3) Ergodic theory: Weeks 9&10 (≈ 4 lectures)

- Stationary sequences, invariant measures -Birkhoff ergodic theorem, Kac recurrence theorem -mutual singularity of ergodic measures, ergodic decomposition via martingale convergence -Subadditive ergodic theorem (only statement and a few applications) -Law of large numbers for iid sequence and martingales

(4) Brownian motion: Weeks 11–14 (≈ 8 lectures)

- Construction of Wiener measure -Sample path properties, zero set -Blumenthal 0-1 law -Strong Markov property, reflection principle -Donsker's theorem -Empirical Distribution and Brownian Bridge, multidimensional Brownian motion and connection to PDE (time permitting)

References

- R. Durrett. Probability Theory and Examples (5th edition)
- P. Billingsly. Probability and Measure
- O. Kallenberg. Foundations of Modern Probability Theory