Math 340 / Stat 231
Advanced Introduction to Probability
Fall 2023

Course syllabus

(Updates in blue. Last updated: 21 Nov.)

Instructor: Nicholas Cook (he/him/his)
nickcook@math.duke.edu

Office hours:
Tuesday 11:30–12:30 in Physics 029A,
Wednesday 12:45–1:55 in Physics 029A and Zoom (hybrid),
Friday 1–2pm in Physics 029A and Zoom (hybrid), or by appointment.

About me: I'm interested in high-dimensional probability, including problems on random matrices, random graphs and large deviations; and related problems in combinatorics, high-dimensional geometry and mathematical physics. I'm starting my 4th year at Duke, after living in California for 10 years, before which I grew up in NC. You can read more about my interests at my webpage.

Email: You can expect a response from me during regular working hours M–F, within one working day. Thus, an email sent to me on Friday may not get a response until the following Monday. (In particular, I cannot monitor my email Friday afternoon for last-minute homework questions!)

Course description. Probability theory is a beautiful area of mathematics that is one of its oldest branches, but continues to be one of the most active areas of research. It spans both pure and applied math, with probabilistic methods playing an increasing role in analysis, number theory and combinatorics; and it provides a foundation and a language for statistics, data science, information theory and statistical physics. It can be used to model very large complicated systems in statistical mechanics, biology and social networks (to name a few) and to shed light on various "universality" phenomena observed in nature.

It is also a pleasure to learn and teach, largely because we all have some intuition for randomness, some of it gained through playing games of chance. In this class we will put probabilistic reasoning in precise mathematical terms, develop some fundamental results and tools such as the law of large numbers and central limit theorem, and use these to study important models such as random walks and Poisson arrivals processes. We'll also see some examples where our intuition might lead us astray.

Prerequisites. Mathematics 202, 212, or 222. Not open to those who have taken Mathematics 230 or Statistics 230.

Comfort with basic rules of set theory (e.g. de Morgan's laws), limits, series, iterated integrals and other material from calculus will be assumed; you may need to brush up on such prerequisite material in your own time as it arises.

Content. This course is an advanced introduction to basic, non-measure theoretic probability covering topics in more depth and with more rigor than MATH 230. Topics include random variables with discrete and continuous distributions, independence, joint distributions, conditional distributions, generating functions, Bayes' formula, random walks; rigorous arguments are presented for the law of large numbers, central limit theorem, and Poisson limit theorems.

Ways this course differs from 230 (i.e. what makes this an "advanced" introduction?):

(1) *In this class we do proofs.* We will approach the subject as mathematicians, not shying away from abstraction and proofs; we do this in pursuit of clarity and precision. We will cover examples to reinforce concepts and consider applications and modeling, but our primary concern is developing the theory of probability. It will not be assumed that you have previously taken a proof-based course. We will also consider limits more often than in 230 (e.g. asymptotics of probabilities as the number of trials n tends to infinity).
(2) *Homework may be challenging.* Math cannot be learned from lectures alone – it is learned by doing! Homework problems will both reinforce and complement lectures. (Really this applies to most subsequent math courses you may take, and college courses in general.) You will learn some new things on the homework – don't expect homework problems to look like small modifications of examples done in the lecture! But you can expect some exam problems to look like homework problems :).

The decision between 340 and 230 isn't simply a question of whether you want to take the "harder" version – it is just as much a matter of taste and fit as it is a readiness for a challenge. *If you have no interest in proofs (and are not open to change!) then Math 230 is a better fit.*

Text: Probability and Random Processes, 4th edition, G. Grimmett and D. Stirzaker. Please make sure you refer to the correct edition.

For Poisson processes we will also use Durrett's Essentials of Stochastic Processes, 3rd ed.(free download through SpringerLink on Duke network)

Course structure.

Class meetings. Will be held in person at the scheduled time of 10:05–11:20am EDT on Tuesdays and Thursdays in Physics 259.

Homework. There will be 12 graded homeworks, due at the end of each week except midterm weeks. They will be due on Fridays at 11:59pm EDT, to be submitted via Gradescope. In Gradescope you must tag the pages of your uploaded assignment with problem numbers. You should make sure your uploaded writeup is legible. "Homework 0", due at the end of first week, is the optional class survey, and submission gets you full credit. HW0 also doubles as a Gradescope practice run.

For homeworks 1–11 you will have all you need to know to complete the homework after the Tuesday lecture of the week the homework is due. You are free to collaborate with other students on the homework, but you must write up your own solutions, *in your own words*, and acknowledge your collaborators at the top (I emphasize: no penalty whatsoever for collaborating!). Collaborating can be a great way to learn (throughout life). Late homework will not be accepted, except as required by the University Policy.

Office hours. I will try to select 3 hours/week so that everyone can attend at least a half hour of one of them. You can also email me to set up an appointment outside of normal office hours. This should be done at least 48 hours in advance to guarantee a meeting (my schedule fills up so more advance notice will help).

In office hours I will generally avoid solving a problem from start to finish in front of you. We may start by discussing what you've tried so far and where you got stuck. I may ask students who have worked a problem out to explain it to others. The point is to promote an open exchange and engage with the material in a way that enhances your learning.

Exams. There will be two midterms and a final exam, in person and closed-book/notes/devices.

- Midterm 1: Oct 5th. Based on the material for homeworks 1–4 (roughly the first 9 lectures).
- Midterm 2: Nov 9th. Based on the material for homeworks 5–8 (roughly through the 18th meeting on Oct 31st). The material builds on itself, so you'll still need to know what was covered by Midterm 1.
- Final exam: Dec 16th from 2–5pm in Physics 259. Will cover all material from the course.

The precise range of material covered by the midterms will be confirmed a week in advance.

Exams will be designed to be straightforward for someone who thoroughly understands the homework problems and important examples/concepts/definitions from the lectures. The best way to prepare for the exams is to put time into the homework. Over-reliance on collaborators to complete the homeworks can lead to worse performance on the exams.

Evaluation: Your grade will be a weighted average of your homework, midterm and final grades, with the following weights: Homework 40%, Midterms $15+15 = 30\%$, Final 30%. Each homework will be graded out of 100%. Your lowest homework score will be dropped, and the remaining 11 homeworks will carry equal weight in your final homework average (that includes the survey!).

Academic integrity. As members of the Duke community, students are expected to uphold the Duke Community Standard.

Technology requirements and policies: Supporting documentation for Gradescope is available here. Technical problems with Gradescope not addressed there must be communicated to me as early as possible. Technical problems with Sakai should be directed to the Duke OIT Service Desk.

Support Services. I understand that college can be a stressful time. Student mental health and wellness are of primary importance at Duke, and the university offers resources to support students in managing daily stress and self-care. Some resources are listed below:

- DuWell provides Moments of Mindfulness (stress management and resilience building) and meditation programming (Koru workshop) to assist students in developing a daily emotional well-being practice. All are welcome and no experience is necessary.
- If your mental health concerns and/or stressful events negatively affect your daily emotional state, academic performance, or ability to participate in your daily activities, many resources are available to help you through difficult times.
- DukeReach provides comprehensive outreach services to identify and support students in managing all aspects of well-being.
- Counseling & Psychological Services (CAPS) services include individual and group counseling services, psychiatric services, and workshops. To initiate services, walk-in/call-in 9:00 AM 4:00 PM (M/W/Th/F) and 9:00 AM 6:00 PM Tuesdays. CAPS also provides referral to off-campus resources for specialized care. Contact: (919) 660-1000
- TimelyCare (formally known as Blue Devils Care) is an online platform that is convenient, confidential, and free way for Duke students to receive 24/7 mental health support through TalkNow and scheduled counseling.

(Tentative and approximate) schedule of class meetings. Some of the listed topics may be explored in homework exercises rather than in the lecture.

Based on Grimmett & Stirzaker 4th Ed, Chapters 1–5, with occasional reference to Chapter 7. Time permitting, selected special topics from Chapters 6–13 may be covered at the end of the course.

(1) Aug 29: Probability spaces: definitions and examples

(2) Aug 31: Probability spaces: further properties and examples
(3) Sept 05: Conditional probability
(4) Sept 07: Independence
(5) Sept 12: Further examples
(6) Sept 14: Random variables and distribution functions
(7) Sept 19: The law of averages (LLNv1)
(8) Sept 21: Discrete and continuous random variables
(9) Sept 26: Random vectors
(10) Sept 28: Discrete RVs: Examples, independence
(11) Oct 03: Discrete RVs: Expectation and variance
(12) Oct 05: Midterm 1 (roughly based on the first 9 class meetings)
(13) Oct 10: Discrete RVs: Expectation, examples
(14) Oct 12: Discrete RVs: Expectation and variance, further examples
(15) Oct 19: Markov's inequality, Chebyshev's inequality
(16) Oct 24: Weak law of large numbers
(17) Oct 26: Discrete RVs: Conditional distribution and expectation
(18) Oct 31: Continuous RVs: Examples, independence, expectation and variance
(19) Nov 02: Continuous RVs: Conditional distribution and expectation
(20) Nov 07: Poisson processes: definition and examples
(21) Nov 09: Midterm 2 (roughly based on class meetings 10–19)
(22) Nov 14: Poisson processes: further properties and examples
(23) Nov 16: Poisson processes: thinning and superposition, conditional distribution of arrivals
(24) Nov 21: Convergence in distribution, CLT, examples
(25) Nov 28: Characteristic functions, inversion and continuity, proof of CLT
(26) Nov 30: Special topics
(27) Dec 05: Special topics
(28) Dec 07: Special topics
(∗∗) Dec 16: Final exam (cumulative)