Math 545: ***Introduction to Stochastic Calculus*** (Spring 2026)

Course syllabus *(Last updated: 9 Jan 2026)*

Instructor: Nicholas Cook (he/him/his)
nickcook@math.duke.edu
Office hours: TBD, or by appointment.
Requests to meet should ideally come at least 2 weekdays in advance as my schedule fills up.
Note that I generally do not address work email on evenings or weekends.

Class meetings: WeFr 8:30–9:45, Physics 119
First meeting: 09 Jan
Last meeting: 22 Apr
∗∗ Class will not be held 27 March

Course description: Introduction to the theory of stochastic differential equations oriented towards topics useful in applications. Brownian motion, stochastic integrals, and diffusions as solutions of stochastic differential equations. Functionals of diffusions and their connection with partial differential equations. Ito's formula, Girsanov's theorem, Feynman-Kac formula, Martingale representation theorem. Additional topics have included one dimensional boundary behavior, stochastic averaging, stochastic numerical methods.

Prerequisites: Real analysis (Math 431/531), probability (Math 230/340).
I do **not** recommend taking the course concurrently with real analysis.
A prior course on measure theory (Math 631 or Stats 711) would be helpful for a deeper understanding of some material but will not be assumed as background.
The first two lectures and the first homework will include a brief review of important elements of probability theory, in particular σ-algebras and conditional expectation.

Primary texts:

- *Introduction to Stochastic Calculus with Applications*, Fima C Klebaner, 3rd edition.

Additional references:

- *Stochastic Differential Equations*, Bernt Øksendal, 6th edition.
- For review of probability theory:
  - *Probability and Random Processes*, Grimmett and Stirzaker
  - *Probability with Martingales*, Williams

Grading: Grades will be based on:

(20%) Biweekly homeworks.
- The lowest score will be dropped.
- Submissions will be done via Gradescope.
- You are welcome (and encouraged!) to collaborate on the homework, but you must list your collaborators at the top of your submission.

(50%) Two in-class midterms (25% each). Tentatively set for 18 Feb and 03 Apr.

(30%) Final presentation (in teams of 3–4). Part of the grade will be based on scribing notes for one or two other presentations. Presentations will happen on April 10, 15, 17, 22.
Topics will be selected from:
- Mathematical biology (Klebaner Ch13)
- Options pricing (Klebaner Ch11–12, Øksendal Ch12)
- Filtering (Klebaner Ch14, Øksendal Ch6)
- Stochastic control (Øksendal Ch11)

Further details forthcoming.

Qualifying credit for the math PhD:. This course can be counted toward the Qualifying requirement for the math PhD program. Please let me know as soon as possible (preferably by the end of the 1st week) whether you plan to qualify. Qualification requires mastery of some material which may not be covered in the lectures, but which you can discuss with me outside of class. See the syllabus for qualification. Qualification will be awarded based on an oral exam at the end of the semester. In borderline cases a more holistic assessment may be made based on overall course performance.