Introduction to Random Matrix Theory

Math 690-40 (Topics in probability), Fall 2025

Instructor: Nicholas Cook (he/him/his)
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Class meetings: TuTh 8:30–9:45, Physics 227.
First meeting: Aug 25. Last meeting: Nov 25

Course description. This course aims for a broad overview of Random Matrix Theory (RMT) covering topics of interest to mathematicians as well as physicists and statisticians. Topics will be selected according to instructor and students' current interests, but will at least cover some core results like Wigner's semicircle law and the Marchenko–Pastur law, the Baik–Ben Arous–Peché (BBP) transition for spiked models, Gaussian ensembles and determinantal point processes, and universality for local spectral statistics. Further possibilities include:

- Elements of free probability theory
- Large deviation principles for the spectrum, HCIZ integrals
- Dyson Brownian motion
- Beta ensembles, Schwinger–Dyson loop equations
- Matrix concentration inequalities
- Community detection in random graphs: stochastic block model, planted clique model
- Connections with physics: genus expansion, quantum chaos, eigenstate thermalization hypothesis, random Schrödinger operators, random band matrices
- Connections with number theory: Montgomery–Dyson pair correlation conjecture for Riemann zeta zeros

Prerequisites. The course aims for a broad audience at the graduate level in math, statistics and physics. The minimal mathematical background is basic probability and linear algebra. Some material will be more accessible with background in graduate-level analysis (Math 631).

Grading. Grades will be based on some combination of:

- Scribing lecture notes once or twice
- A final group presentation on a paper
- and/or: completion of some optional exercises. Students can choose from a range of exercises based on their background. For students with less background in mathematical analysis there will be options to run numerical simulations.

References.

- *Topics in random matrix theory*, Terence Tao
- *An Introduction to Random Matrices*, Anderson, Guionnet and Zeitouni.
- *Free Probability and Random Matrices*, Mingo and Speicher
- *High-dimensional Probability*, Roman Vershynin