

Math 490: Topics in the Clay Millennium Prize Problems

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New course offered in Spring 2025

A Number Theory RTG Initiative

Overview

In 2000, the Clay Mathematics Institute established a prize of one million dollars for each of seven Millennium Problems, as an indication of their central importance in current mathematical research. This followed a tradition established in 1900 by David Hilbert, who formalized a list of 23 open problems that then motivated influential research programs in mathematics over the subsequent century. This topics course for advanced undergraduate mathematics students describes a selection of three of the seven Millennium Prize Problems.

Topics in spring 2025

In spring 2025, Math 490 will focus on three Millennium Problems:

1. The Navier-Stokes Equation problem, which asks for the proof of the existence of smooth solutions to a partial differential equation that governs the motion of a fluid in 3 dimensions;
2. The Riemann Hypothesis, a conjecture about a complex-variable function in number theory that predicts how the precise distribution of prime numbers deviates from their average distribution;
3. The Hodge Conjecture, which determines how much of the topology of the solution set of a system of algebraic equations can be defined in terms of further algebraic equations.

Students in the course will learn rigorous introductory material in the mathematical area of each problem, and gain the ability to describe precisely what the problem is, what its impact would be if solved, and some indications of why it is difficult to solve.

Minimal required pre-requisites

- A course on linear algebra (such as Math 221)
- A course on vector (or multi-variable) calculus (such as Math 222)
- A course on complex analysis (such as Math 333)

Advantageous background

Some background in topology, differential geometry, ODE, or number theory would be helpful, but is not required.