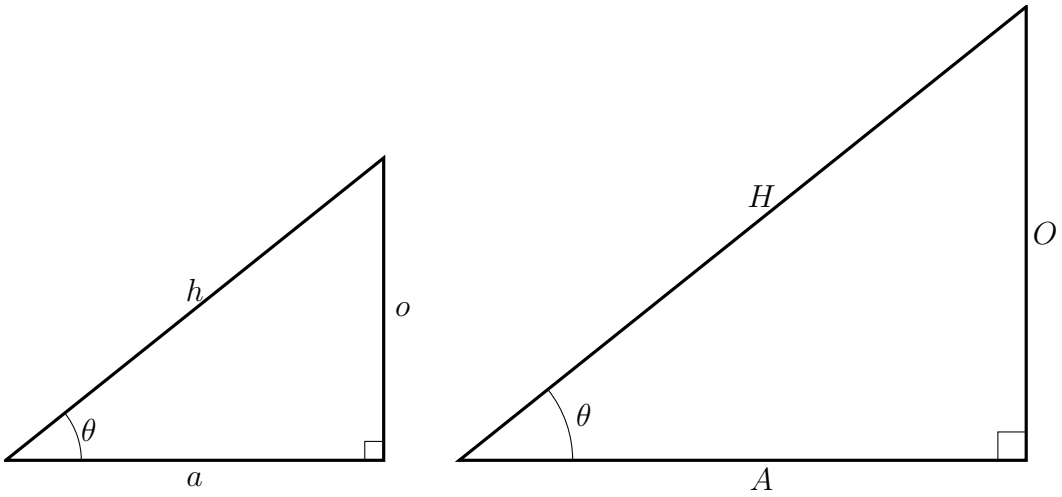
Right-Angled Triangles

Consider two right-angled triangles with one identical angle (other than the right angle):



- What can you say about the third angle in each of the triangles?.
- Therefore, the two triangles are _____.
- This implies that:

$$\frac{o}{h} = \_\_\_\_, \text{ and } \frac{a}{h} = \_\_\_\_, \quad \frac{o}{a} = \_\_\_\_.$$

Conclusion: The above ratios only depend on ___.

Definitions - Basic Trig Functions: given an angle θ in a right-angled triangle, we define the following three functions:

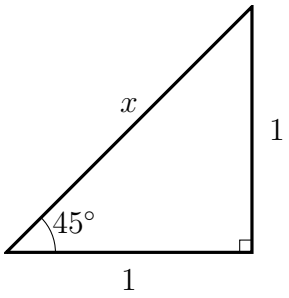
$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \ \cos\theta = \frac{\text{adj}}{\text{hyp}}, \ \tan\theta = \frac{\text{opp}}{\text{adj}},$$

where opp, and adj are the lengths of the sides of the corresponding right-angled triangle positions opposite, and adjacent to angle θ respectively, and hyp is the length of the hypotenuse.

For what angles are these functions currently defined? i.e. What are their domains?

Special Values of the Trig Functions

By finding the value of x in the following $45° - 45° - 90°$ triangle exactly (no decimals!), compute the values below:

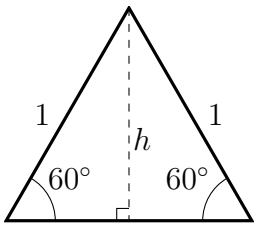


$\sin 45° =$ _____

$\cos 45° =$ _____

$\tan 45° =$ ____

By finding the value of h (i.e. the length of the dashed line) in the following $60° - 60° - 60°$ triangle exactly (no decimals!), compute the values below:



$\sin 60° =$ _____

$\cos 60° =$ ____

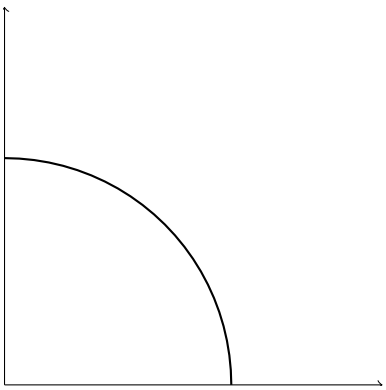
$\tan 60° =$ _____

You can also use the same triangle to compute the following values (rotate the page 90°):

$$\sin 30° = \_\_\_\_,\ \cos 30° = \_\_\_\_\_,\ \tan 30° = \_\_\_\_\_.$$

Extending the Domains

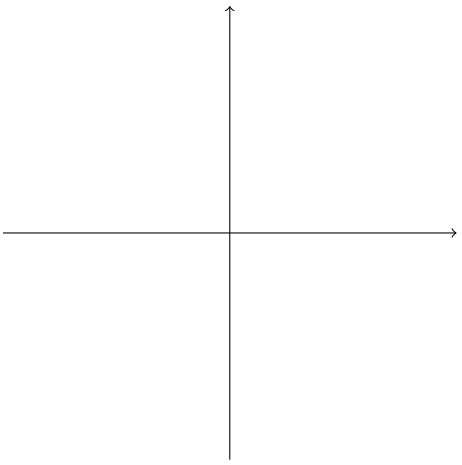
Suppose that the hypotenuse of a right-angled triangle has length 1. Draw such a triangle on the axes to the right, with its angle θ located at the origin, and the adjacent edge on the x-axis. Imagine the angle increasing from 0° to 90°. Why does it trace out a quarter cirlce?



Now fix the angle θ, and label the corresponding point on your traced shape (x, y). Then

$$\cos\theta = \_\_\_\_, \text{ and } \sin\theta = \_\_\_\_.$$

Continue drawing your shape all the way around on the next set of axes. Label an angle with $90° < \theta < 180°$.



For such an angle, we *define* $\cos\theta = x$, and $\sin\theta = y$, where x and y are the coordinates of the point on the unit circle corresponding to the angle θ, as measured anti-clockwise from the positive horizontal axis.

For angles $90° < \theta < 180°$, is $\sin\theta$ positive or negative? What about $\cos\theta$?

Definitions - Trig Functions for General Angles

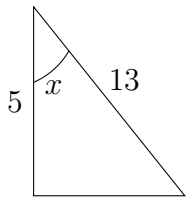
Given any angle θ, $\sin\theta$ is the y-coordinate of the point on the unit circle whose corresponding radius makes the angle θ with the positive horizontal axis, measure anti-clockwise. $\cos\theta$ is the x-coordinate of the same point. To get negative angles, measure clockwise instead.

For an angle θ, and the corresponding point on the unit circle (x, y),

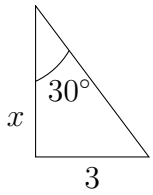
$$\tan\theta = \frac{\quad}{\quad} = \frac{\quad}{\quad}.$$

Homework: Solving Triangles and Word Problems

1. Given the right triangle to the right, find the *exact* values of $\sin x$, $\cos x$, and $\tan x$. □Regrade



2. Given the right triangle to the right, solve for x. □Regrade



For all the problems below, draw a good picture first, then solve.

3. A rocket is fired at sea level and climbs at a constant angle of 75° through a distance of 10,000 feet. Approximate its altitude to the nearest foot. □Regrade

4. An airline pilot wishes to make his approach to an airstrip at an angle of 10° with the horizontal. If they are flying at an altitude of 5000 feet, at what ground distance from the airstrip should they begin their descent? □Regrade

5. A 16 foot long ladder is leaning against a wall and making a 60° angle with the ground. □Regrade Without using your calculator determine exactly how high on the wall the top of the ladder is resting.

6. An astronomer is studying two distant stars each approximately 12 thousand light □Regrade years from earth. They find that the angle spanned by the two stars, with the earth at its vertex, is approximately 74°. Estimate the distance between the two stars.

7. From a point A that is 8 meters above level ground, some distance away from a □Regrade building, the angle of elevation of the top of the building is 31° and the angle of depression of the base of the building is 12°. Approximate the height of the building. (Hint: you will need two equations, as you have two unknowns!)