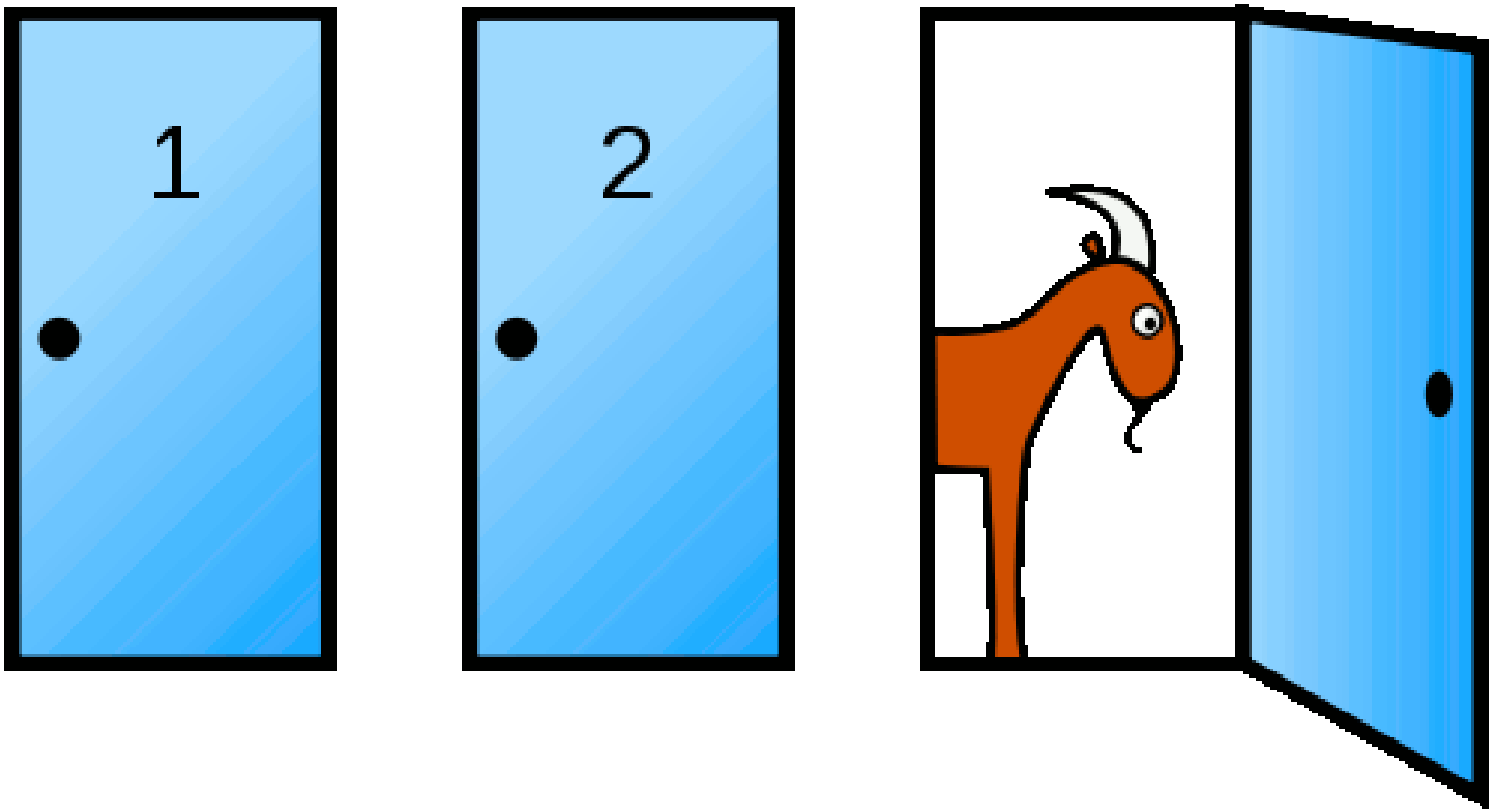
Truth is stranger than fiction: A look at some improbabilities

Rick Durrett


Monty Hall!

You picked door 1, should you switch?



Marilyn vos Savant is an American magazine columnist, author, lecturer and playwright who rose to fame through her listing in the Guinness Book of World Records under "Highest IQ". Since 1986 she has written Ask Marilyn, a Sunday column in Parade magazine in which she solves puzzles and answers questions from readers on a variety of subjects.

Her Sept. 9, 1990 column was devoted to the Monty Hall problem. Vos Savant answered arguing that the selection should be switched to door #2 because it has a 2/3 chance of success, while door #1 has just 1/3.

Reaction to Marilyn vos Savant's

You blew it, and you blew it big! Since you seem to have difficulty grasping the basic principle at work here, I'll explain. After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your selection or not, the odds are the same. There is enough mathematical illiteracy in this country, and we don't need the world's highest IQ propagating more. Shame!

Scott Smith, Ph.D.
University of Florida

See marilynvossavant.com for the original column and many of the letters.

Solution to Monty Hall

Suppose #1 is chosen.

	#1	#2	#3	host's action
case 1	donkey	donkey	car	opens #2
case 2	donkey	car	donkey	opens #3
case 3	car	donkey	donkey	opens #2 or #3

$P(\text{case 2}, \text{open door \#3}) = 1/3$ and

$$P(\text{case 3}, \text{open door \#3}) = P(\text{case 3})P(\text{open door \#3}|\text{case 3}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

$P(\text{open door \#3}) = 1/3 + 1/6 = 1/2$ so

$$P(\text{case 3}|\text{open door \#3}) = \frac{P(\text{case 3}, \text{open door \#3})}{P(\text{open door \#3})} = \frac{1/6}{1/2} = \frac{1}{3}$$

Easier Solution

Your probability of winning was 1/3 when you picked and it didn't change when Monty opened door 3.

Cognitive Dissonance in Monkeys

Yale psychologists measured monkeys preferences by observing how quickly each monkey sought out different colors of M&Ms. In the first step, the researchers gave the monkey a choice between say red and blue. If the monkey chose red, then it was given a choice between blue and green. **Nearly two-thirds of the time** it rejected blue in favor of green, which seemed to jibe with the theory of choice rationalization:

"once we reject something, we tell ourselves we never liked it anyway."

Who's the monkey?

There are six possible orderings:

RGB	*BRG*	*GRB*
RBG	*BGR*	*GBR*

In three of these (in red) $R > G$ and in 2/3's of these $B > G$.

Observation of economist M. Keith Chen.

Story from New York Times, April 2008.

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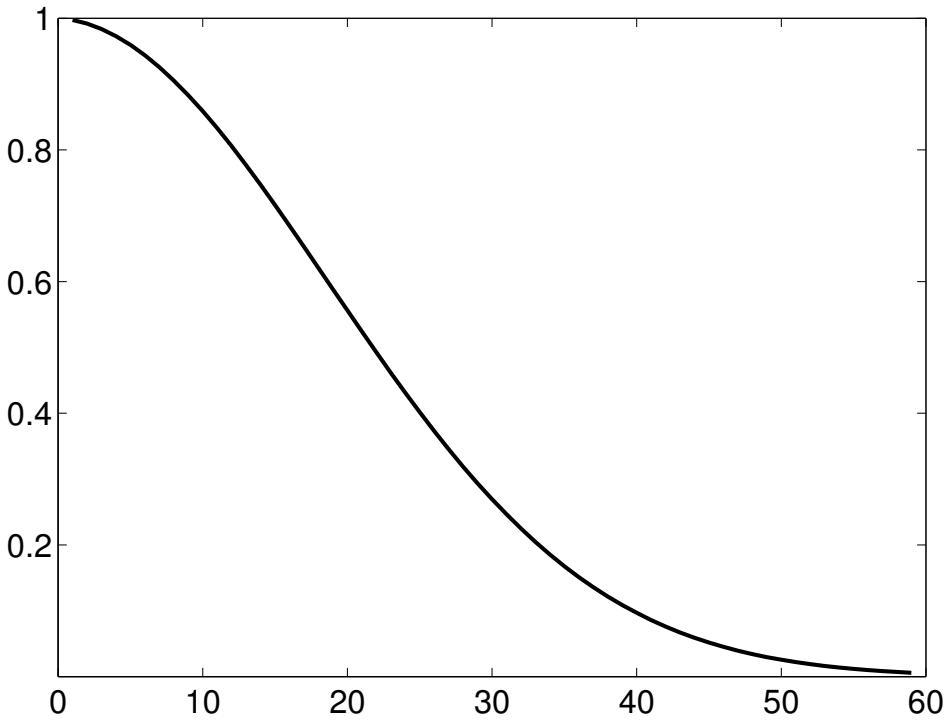
A match is likely because there are $(40 \cdot 39)/2 = 780$ pairs of players that share a birthday with probability $1/365$. Probability of no match is about 0.11.

Probability all birthdays different for n people

$$\frac{365 \cdot 364 \cdots 366 - n}{(365)^n}$$



Atlanta Braves

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The probability of three double birthdays is 0.1796

$$\frac{1}{3!} \cdot \frac{40!}{(2!)^3 34!} \frac{365 \cdot 364 \cdots 329}{(365)^{40}}$$

Probability two players born on April 24: .005275. Expected number of days with two births is 1.925

P(triple birthday in the Senate) ?

Challenge: calculate the probability that in a group of 100 people there is at least one triple birthday.

Approximate solution: The number of senators born on a given day is approximately Poisson with mean 100/365.

Probability of triple $= e^{-100/365}(100/365)^3/3! = 0.002606$, expected number among 365 days $= 0.9512$

Probability of double $= e^{-100/365}(100/365)^2/2! = 0.028536$, expected number among 365 days $= 10.4$

Answers for this Senate

One triple birthday May 3: Jim Risch (Idaho), David Vitter (Louisiana), Ron Wyder (Oregon)

Thirteen double birthdays: Jan 7, 10, 23; March 31, June 18, 22, August 24, Sept 29, October 20, November 17, 23, December 7, 28

A Birthday Triple

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Complication. Labor was induced for the third child on 10/9/10.

Pick 4 coincidence

To quote a United Press story on September 10, 1981:

> *"Lottery officials say that there is 1 chance in 100 million (10^8) that the same four digit lottery number would be drawn in Massachusetts and New York on the same night. That's just what happened Tuesday. The number 8902 came up paying $5842 in Massachusetts and $4500 New York."*

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Some number will be picked in MA.
NY will match with probability 10^{-4}

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2222. In April 20, 2013 these numbers came up in the North Carolina Pick 4 lottery. 933 poeple won $5000 for a $1 bet, and 1118 won $2500 for a 50 cent bet.

If all numbers were played with the same probability, NC would need to sell 20,000,000 tickets to have 2000 winners.

Sally Clark

In 1999, a British jury convicted Sally Clark of murdering her two children who had died suddenly at the ages of 11 and 8 weeks respectively of sudden infant death syndrome or "cot deaths". There was no physical or other evidence of a murder, nor was there a motive. Most likely the jury was convinced by a pediatrician who said that a baby had a probability of roughly 1/8500 of dying a cot death, so having two children die this way had probability roughly 1/73,000,000.

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Sally Clark spent 3 years in jail before the conviction was overturned

Lottery Double Winner

A New Jersey woman, Evelyn Adams, won the lottery twice within a span of four months raking in a total of 5.4 million dollars. She won the jackpot for the first time on October 23, 1985 in the Lotto 6/39 in which you pick 6 numbers out of 39. Then she won the jackpot in the new Lotto 6/42 on February 13, 1986. Lottery officials calculated the probability of this as roughly one in 17.1 trillion.

$$\frac{1}{C_{39,6}} \cdot \frac{1}{C_{42,6}} = \frac{1}{17.1 \times 10^{12}}$$

$C_{n,m} = n!/m!(n-m)!$ is the number of ways of picking m things out of n.

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Many people who play the lottery buy more than one ticket. Suppose 1,000,000 people buy 5 tickets each.

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Many people who play the lottery buy more than one ticket. Suppose 1,000,000 people buy 5 tickets each.

Probability is now about 1/200. Now take into account the number of states with lotteries.

Maureen Wilcox.

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Unfortunately for her, her Massachusetts numbers won in Rhode Island and vice versa.

Scratch-off Triple Winner.

81-year old Keith Selix won three lottery prizes totaling $81,000 from scratch off games in the twelve months preceding May 3, 2006. He won $30,000 twice in "Wild Crossword" games and $21,000 playing "Double Blackjack." The odds of winning in these games are 89,775 to 1 and 119,700 to 1 respectively.

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One of the reasons Selix won so many times in 2006 is that he spent about $200 a week or more than $10,000 a year on scratch-off games. Expected number of wins $= 10^4/10^5 = 0.1$, so the probability of exactly three wins would be

$$e^{-0.1}\frac{(0.1)^3}{3!} \quad \text{or} \quad < \frac{1}{60,000}$$

Luckiest Gas Station in America (Bishop, TX)

Scratch-off Quadruple Winner

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Joan got her Ph.D. in Statistics from Stanford. An article in Harper's suggested that she may have figured out the random number generator that makes the tickets.

Ginther lives in Las Vegas but spends two months a year in Bishop Texas playing scratch off tickets.

The lucky store was shut down by the IRS this year once they discovered the store owner was holding boxes of high stakes scratch off tickets for her.

Winning the lottery without cheating

March 30, 2005. In the 29 state powerball lottery there are an average of 4-5 second prize winners who got five of the six numbers right. However there were 110 people who had the numbers 22, 28, 32, 33, 39.

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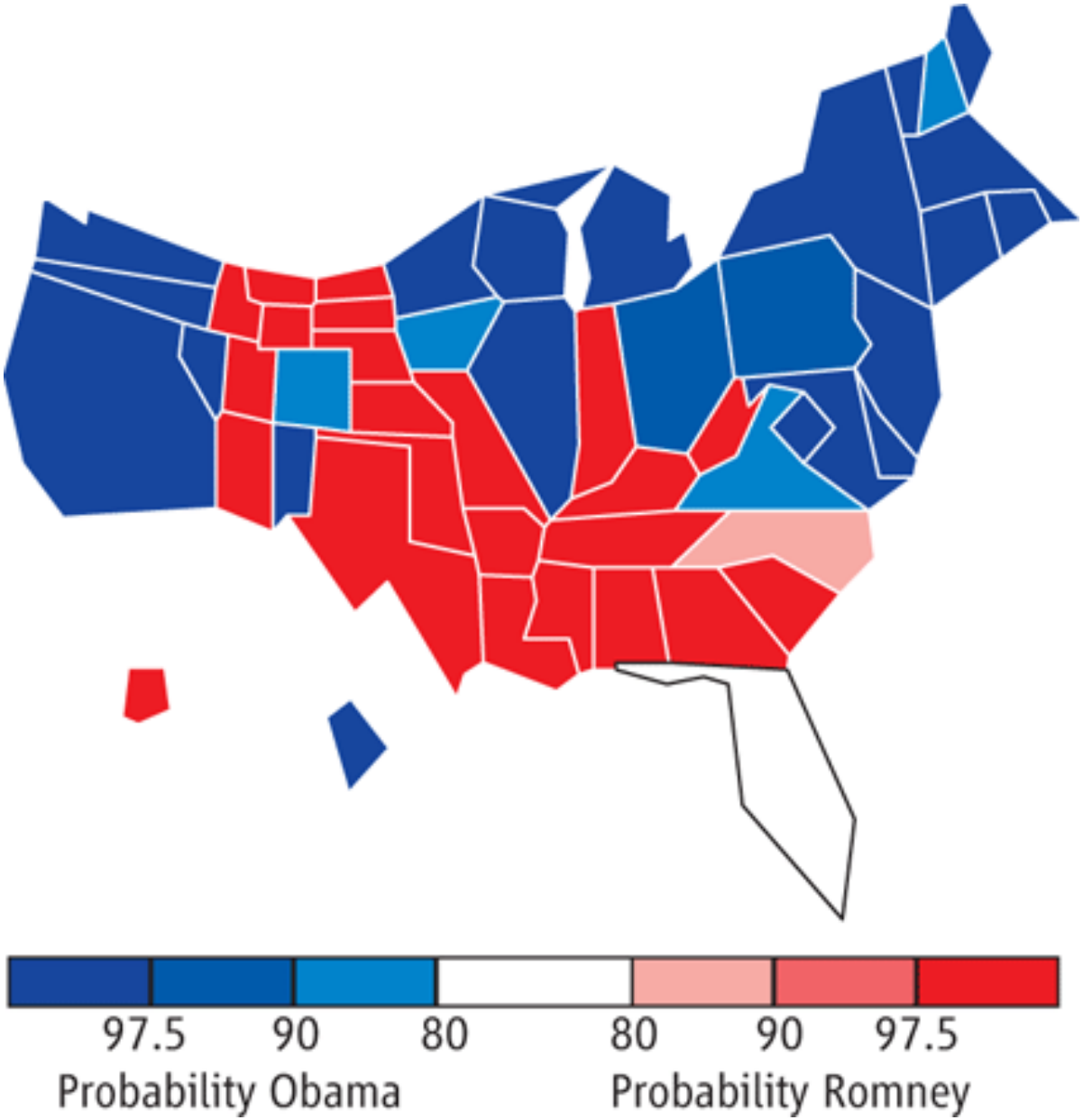
The players got their numbers inside fortune cookies that came from the same factory in Long Island City, Queens.

Sometimes Things are not Surprising

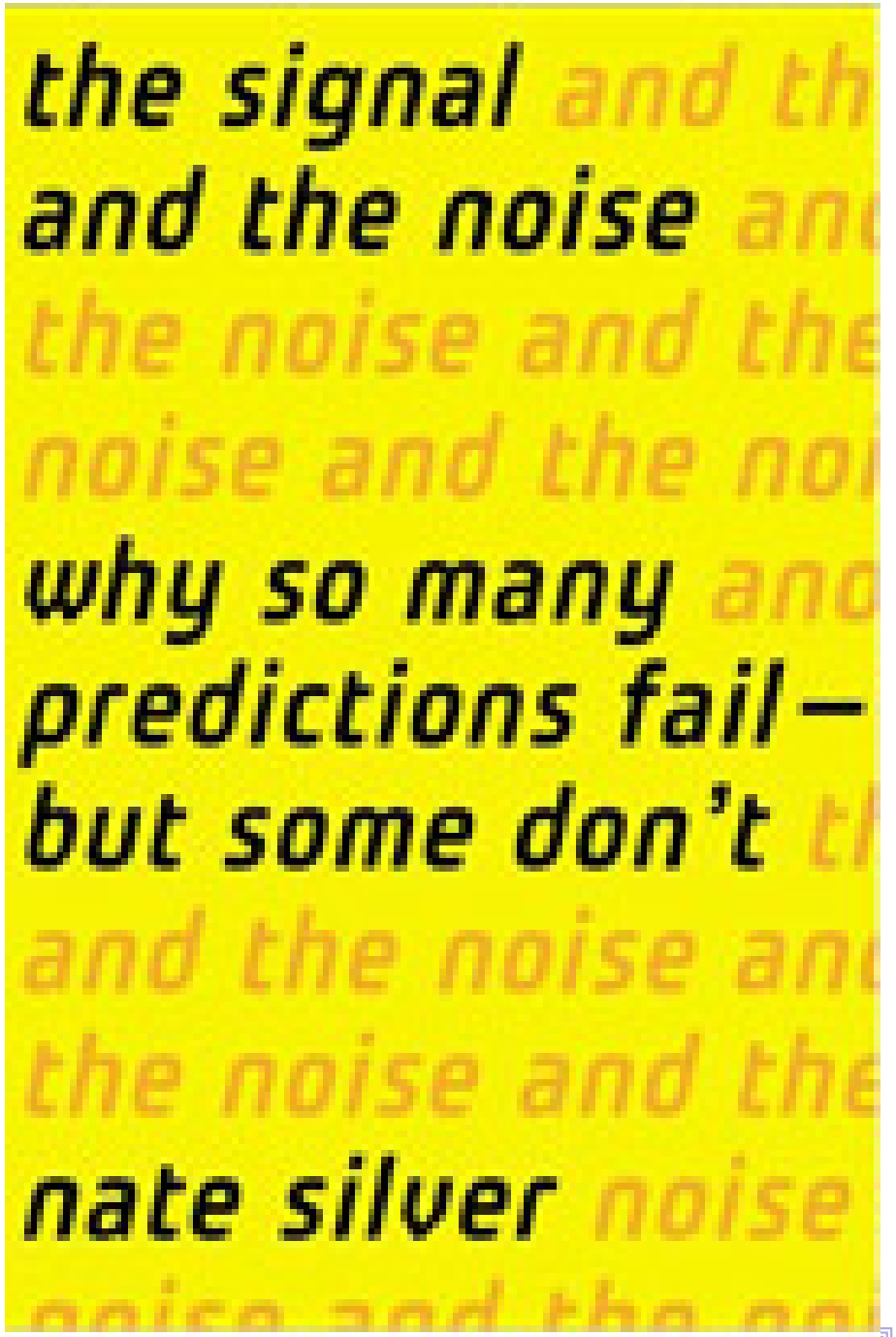


Nate Silver's 538.com on Monday 11/5/2012

State	P(Obama wins)
Nevada	.934
Ohio	.906
N.H.	.846
Iowa	.843
Colorado	.797
Virginia	.794
Florida	.503
N. Carolina	.256

Obama 313 electoral voters, P(Obama wins election) = .909

Princeton Election Consortium on that Monday: p = .992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Every day 30 events of probability 1/10,000,000 happen to someone in the U.S. Of course it would be surprising if one of these happened to you.