Programmable Matter and Emergent Phenomena

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.

Collectives at various scales

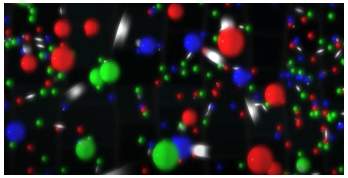
People

Biology

Robots

Tiny Robots

Particles



Questions:

- What can they do **collectively**?
- What type of **computation**?
  Communication? **Memory**?
- Can we **understand / program** it?
- How **predictable** is the behavior?

Swarm Robotics / Programmable Matter

(as Self-Organizing Particle Systems)

Active Matter: ensemble of self-organizing computational "particles"

+

Programmable to change their collective physical properties

Swarm Robotics / Programmable Matter
(as Self-Organizing Particle Systems)

Active Matter: ensemble of self-organizing computational "particles" + Programmable to change their collective physical properties

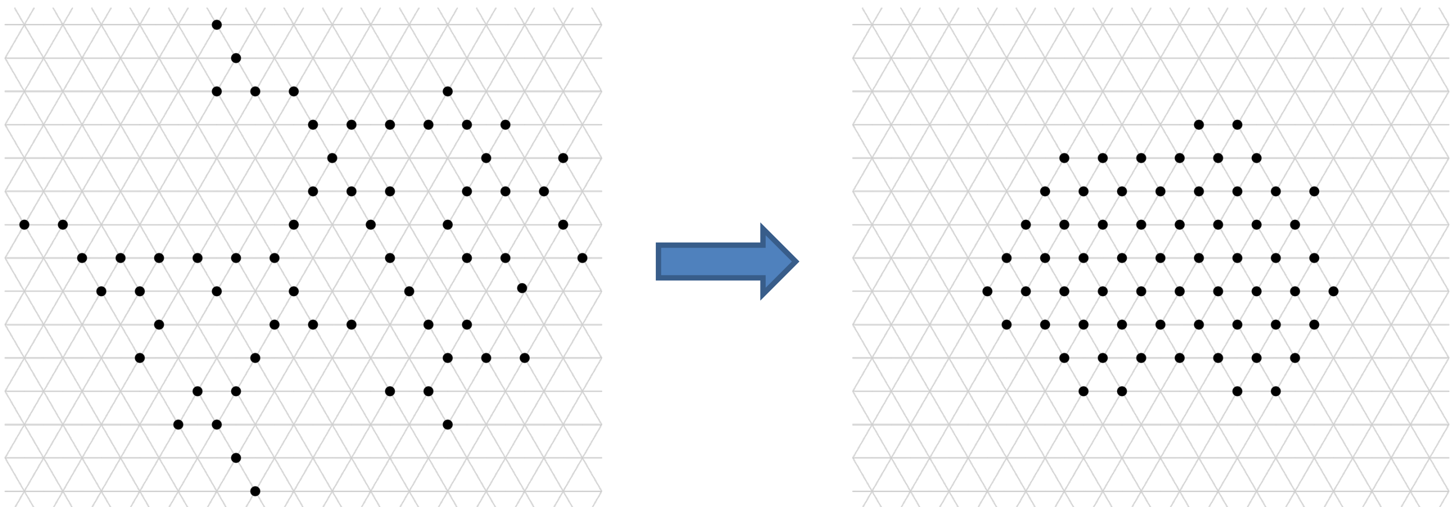
Algorithms: Devise the local, distributed rules that each particle runs in order to achieve the desired emergent, collective behavior

- No human intervention or central control
- Scalable
- Indistinguishable particles
- Oblivious to global properties

E.g. Compression

Question: Using local, distributed rules, how can particles "compress" (or "aggregate")? *Assume particles are simply connected.

Triangular lattice



Not compressed

Compressed

Def: A configuration is α-compressed if its perimeter is at most α times the minimum perimeter (for this number of particles).

Compression Algorithm

[Cannon, Daymude, R., Richa '16]

A distributed, stochastic algorithm for compression:

- Ensures system connectivity on the triangular lattice.
- Poisson clocks to activate particles (i.e., no synchronization).
- Metropolis probabilities to converge to $\pi(\sigma) \propto \lambda^{e(\sigma)}$, for $\lambda > 1$.

Fix $\lambda > 1$. Start in any connected configuration.

When a particle activates (according to its Poisson clock):

- Pick a random neighboring node.
- If the proposed node is unoccupied, and certain properties hold*,

  move with probability $\min\{\lambda^{\Delta e}, 1\}$.

- Otherwise, do nothing.

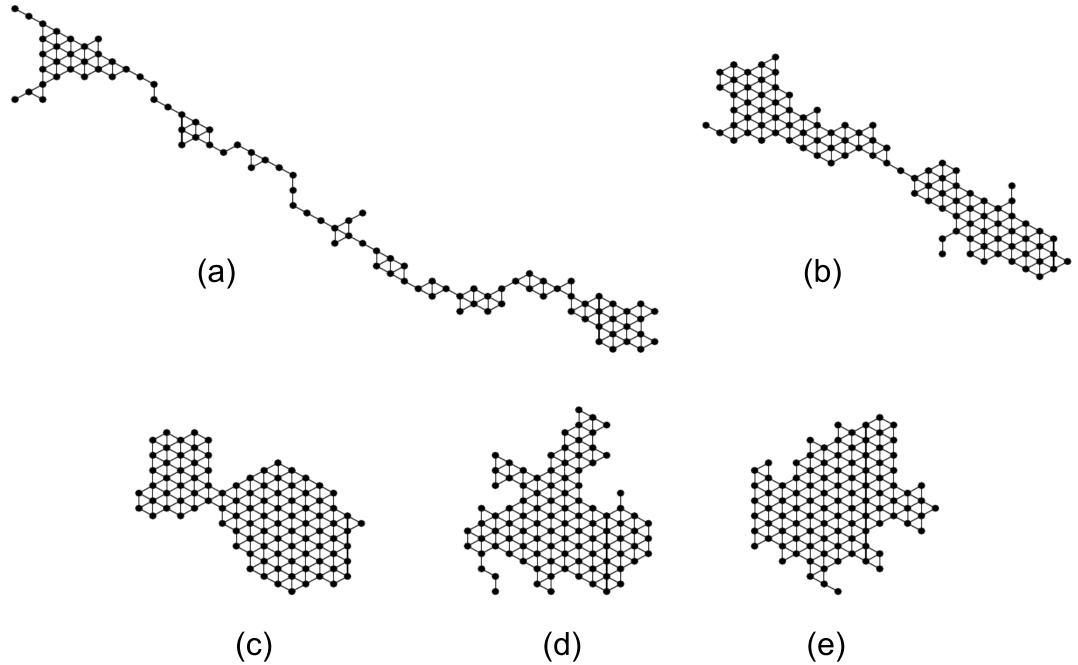
*To maintain connectivity.

Compression Simulations

λ = 4

100 particles after:
a) 1 million
b) 2 million
c) 3 million
d) 4 million
e) 5 million
iterations.

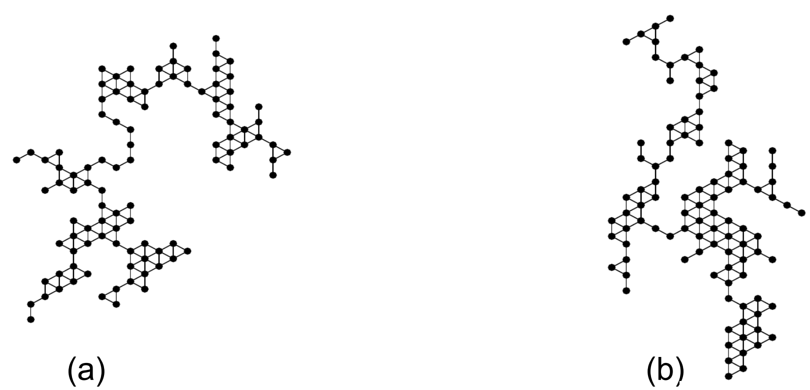
Compression.



λ = 2

100 particles after:
a) 10 million
b) 20 million
iterations.

No compression.



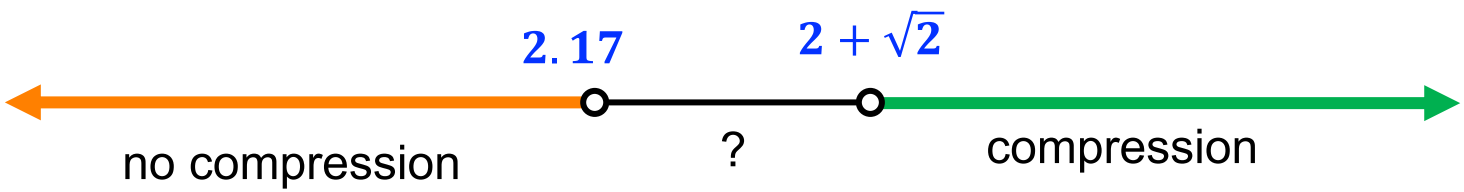
Compression: Theorems

[Cannon, Daymude, R., Richa '16]

Defn: A configuration is α-compressed if its perimeter is at most α times the minimum perimeter.

Thm: For all $\lambda > 2+\sqrt{2}$, there exists $\alpha = \alpha(\lambda)$ s.t. particles are α-compressed at stationarity almost surely.
(E.g., when $\lambda = 4$, $\alpha = 9$.)

Thm: When $\lambda < 2.17$, for *any* $\alpha > 1$, the probability particles are α-compressed at stationarity is exponentially small.



Note: Expansion works similarly for small λ.

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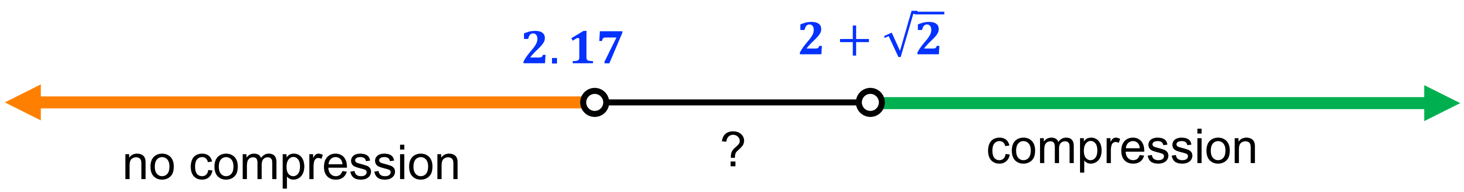


Note: Expansion works similarly for small λ.

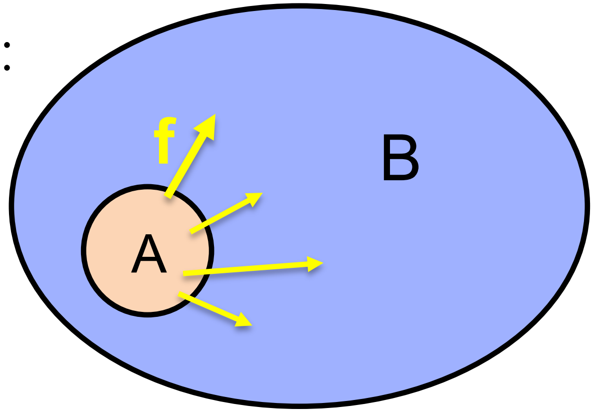
Main proof technique: Peierls Argument

To show that some subset has exponentially small probability:

Define $f : A \rightarrow B$ so that, for $c_1 > c_2 > 1$ such that:

- For all $a \in A$, $\pi(a)\, e^{c_1 n} < \pi(f(a))$
- For all $b \in B$, $|\{f^{-1}(b)\}| < e^{c_2 n}$

Then: $\pi(A) < e^{(c_2 - c_1)\, n} << 1.$



Physics: Distinguish Gibbs states;

Comp Sci: Bound mixing times through identifying small cutsets;

Active Matter: Bound likelihood of (un)desirable ensemble behavior.

Proof Techniques

Thm: For all $\lambda > 2 + \sqrt{2}$, there exists $\alpha = \alpha(\lambda)$ s.t. particles are α-compressed at stationarity almost surely.

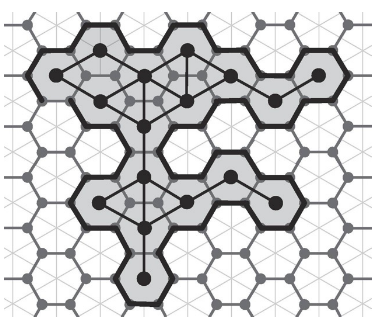
Pf: Note $p(\sigma) = 3n - e(\sigma) - 3$, so we can express the stat. dist'n as:

$$\pi(\sigma) \propto \lambda^{e(\sigma)} = \lambda^{-p(\sigma)}/Z$$

Let S_α = configurations with perimeter > $\alpha\, p_{min}$
m_k = number of configurations with perimeter k.

$$\pi(S_\alpha) = \sum_{k = \alpha\, p_{min},}^{p_{max}} m_k\, \lambda^{-k} / Z$$

The (# configs with perim. k) < (# SAWs in the hexagonal lattice), i.e.,



$|\{\text{SAWs of length } t\}| \sim (\mu_{hex})^t = (2 + \sqrt{2})^{t/2}$

[Duminil-Copin and Smirnov '12]

Proof Techniques

Thm: For all $\lambda > 2+\sqrt{2}$, there exists $\alpha = \alpha(\lambda)$ s.t. particles are α-compressed at stationarity almost surely.

Pf: Let $S_\alpha =$ configurations with perimeter > $\alpha\, p_{min}$
$m_k =$ number of configurations with perimeter k.

$$\pi(S_\alpha) = \sum_{k=\alpha p_{min}}^{p_{max}} m_k \lambda^{-k} / Z \quad \lesssim \quad \sum_{k=\alpha p_{min}}^{p_{max}} (2+\sqrt{2})^{k+5/2} \lambda^{-k} / Z$$

.... which is exponentially small for $\boldsymbol{\lambda} > \mathbf{2}+\sqrt{\mathbf{2}}$.

Thm: When $\lambda < 2.17$, for *any* $\alpha > 1$, the probability particles are α-compressed at stationarity is exponentially small.

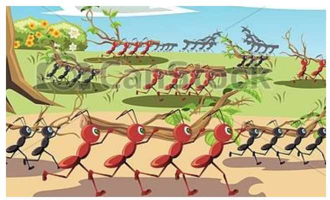
Another Peierls argument to show non-compression whp using bijection between configs and hydrocarbons (or "animals" on the hex lattice).

More Self-Organization in Nature

Compression

Bridging

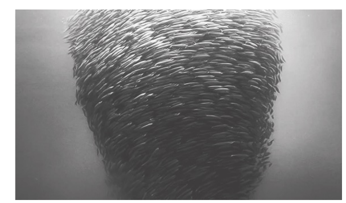
Separation



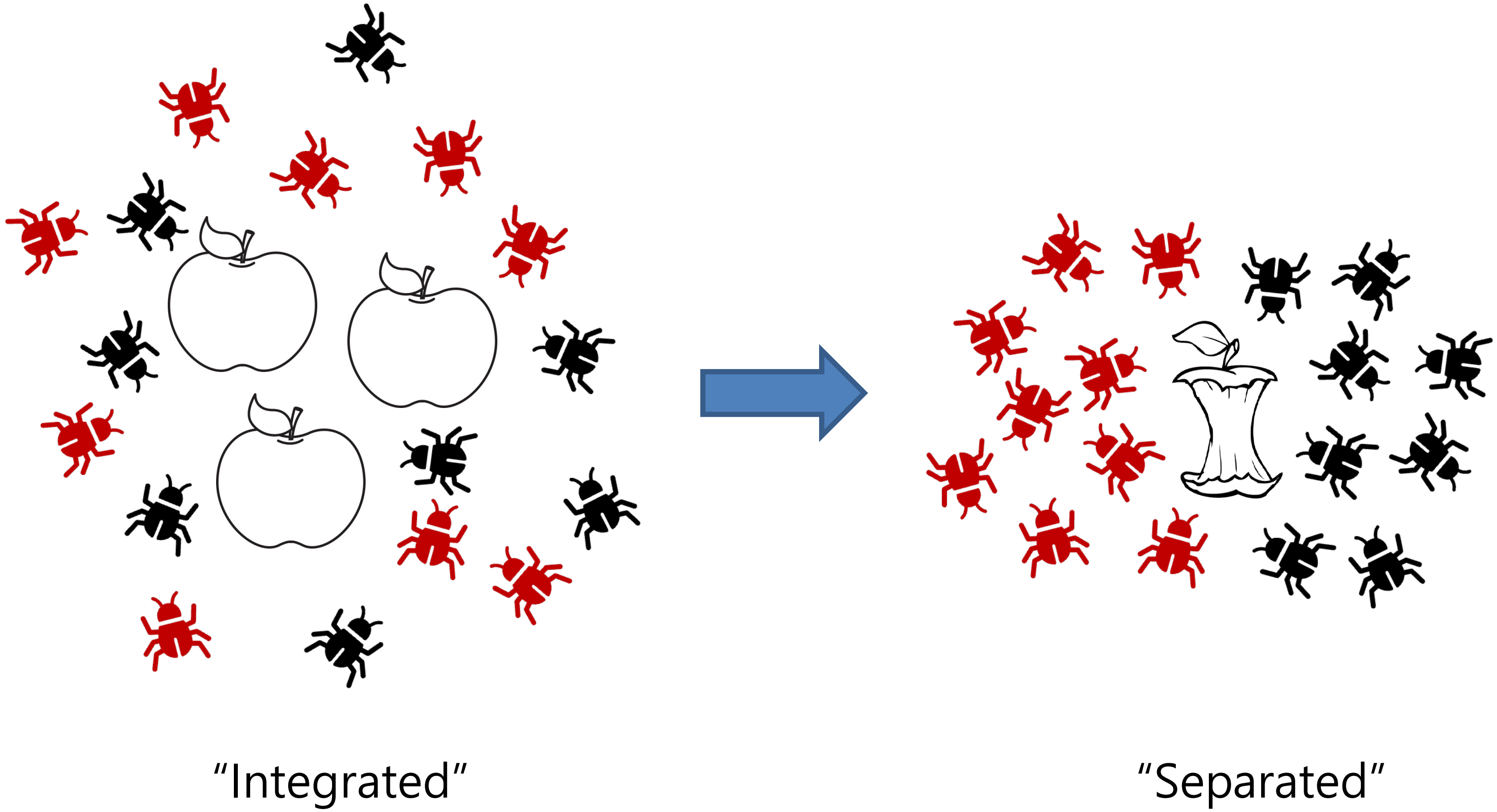
Locomotion

Alignment

Flocking

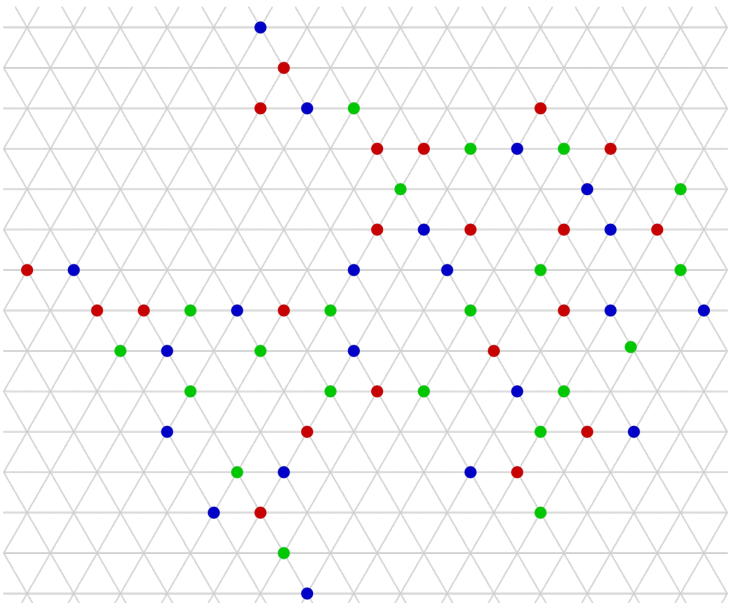


Separation (or Speciation)

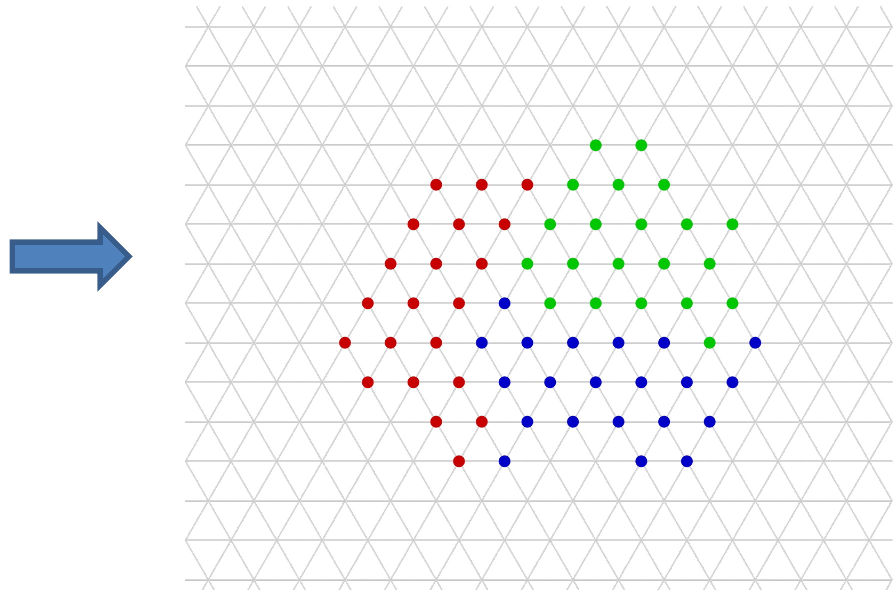


Separation

Question: Using local, distributed rules, how can heterogeneous particles "compress" overall while also "separating" into (mostly) monochromatic groups?



Neither compressed nor separated

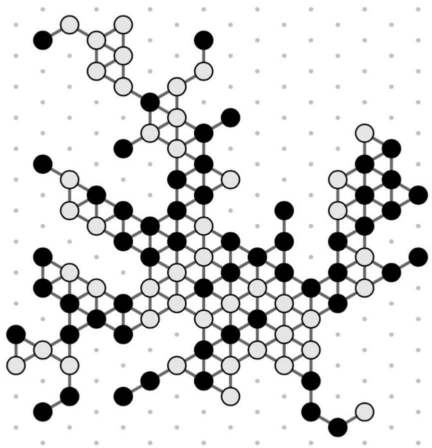


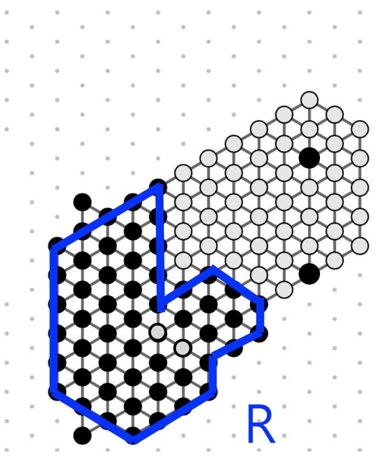
compressed and separated

Definition of Separated

Defn: A configuration is (β,δ)-separated if there is a subset of particles R s.t.:

1. There are at most $\beta\sqrt{n}$ particles with exactly one endpoint in R;
2. The density of particles of color c_1 *inside* R is at least $1-\delta$;
3. The density of particles with color c_1 *outside* R is at most δ.





Compressed and separated

MC for Separation

Distributed algorithm for separation:

- Ensures global connectivity and is not synchronized.
- Uses Metropolis probabilities to converge to:

$$\pi(\sigma) \propto \lambda^{e(\sigma)} \cdot \gamma^{m(\sigma)},$$

for bias parameters λ (for compression) and γ (for separation), where $m(\sigma)$ is the # of monochromatic edges.

Fix λ and γ. Start in any connected configuration.

When a particle activates (according to its Poisson clock):

- Pick a random neighbor.
- Move with probability $\min\{(\lambda^{\Delta e} \cdot \gamma^{\Delta m}), 1\}$.
- Otherwise, do nothing.

Separation for large γ

Stationary distribution: $\pi(\sigma) \propto \lambda^{e(\sigma)} \cdot \gamma^{m(\sigma)} = (\lambda\gamma)^{-p(\sigma)} \cdot \gamma^{-h(\sigma)}$.

Thm: When $\lambda\gamma > 6.83$ and $\gamma > 5.66$, there exists α s.t. the particle system is α-compressed and separated at stationarity a.s.

Have to account for both *monochromatic / heterogenous* edges!

Proof uses the *cluster expansion* + a *Peierls arg*.

Thm: When $\lambda(\gamma + 1) > 6.83$ and $0.98 \le \gamma \le 1.02$, there exists $\alpha(\lambda, \gamma)$ s.t. the particle system will be α-compressed and integrated (i.e., not separated) at stationarity a.s.

Uses the *high temperature expansion* (to express Z as a weighted sum even degree subgraphs):

$$\sum_{\sigma \in \Omega_\Lambda} \gamma^{-h(\sigma)} = (\ldots) \sum_{\text{even } E \subseteq E(\Lambda)} \left(\frac{\gamma - 1}{\gamma + 1}\right)^{|E|}$$

+ a similar strategy with the *cluster expansion* + a *Peierls arg*.

Strategies for Collective Behaviors

Local rules → ? → *Global properties*

- Carefully choreograph interactions
- Define a useful potential function
- Infer global behavior from a meaningful stationary dist'n

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Driving Collective Behavior

Local rules → ***Global properties***

Boltzmann Distribution

$$\Pi(\sigma) = e^{-\beta H(\sigma)}/Z$$

Driving Collective Behavior

Local rules

Global properties

For a desirable set $A \subseteq \Omega$,

$\Pi(A) \geq 1 - e^{-cn}$

Boltzmann Distribution

$$\Pi(\sigma) = e^{-\beta H(\sigma)}/Z$$

Peierls Argument

Driving Collective Behavior

Local rules

$P(\sigma,\tau) = \min(1, \Pi(\tau)/\Pi(\sigma))$

Global properties

For a desirable set $A \subseteq \Omega$,

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Boltzmann Distribution

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Metropolis-Hastings Algorithm

Peierls Argument

Driving Collective Behavior

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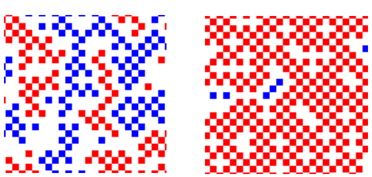
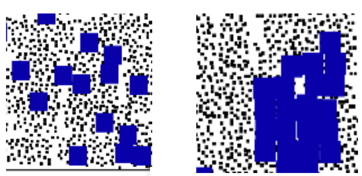
Peierls Argument

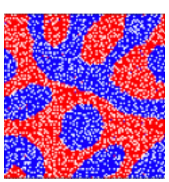
Particle/spin systems



Colloids

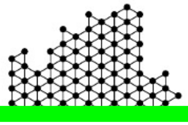


Schelling Segregation



Local rules → Emergent behavior

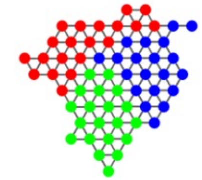
Collective Behavior



Emergent behavior → Local rules

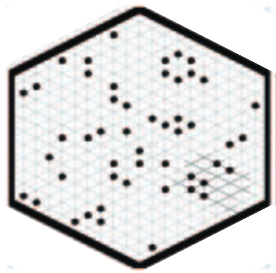
Stochastic Distributed Algorithms for Collectives

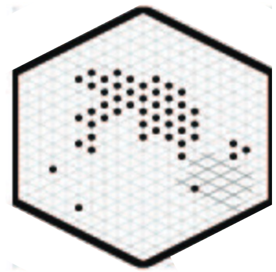
- Separation / Integration [Cannon, Daymude, Gokmen, R., Richa, ‘18]



- Aggregation / Dispersion [Li, Dutta, Cannon, Daymude, Avinery, Aydin, Richa, Goldman, R. ’21]

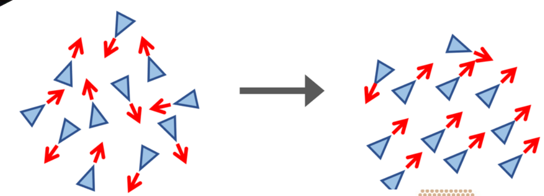
Dispersion



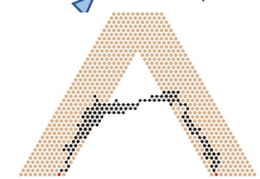


Aggregation

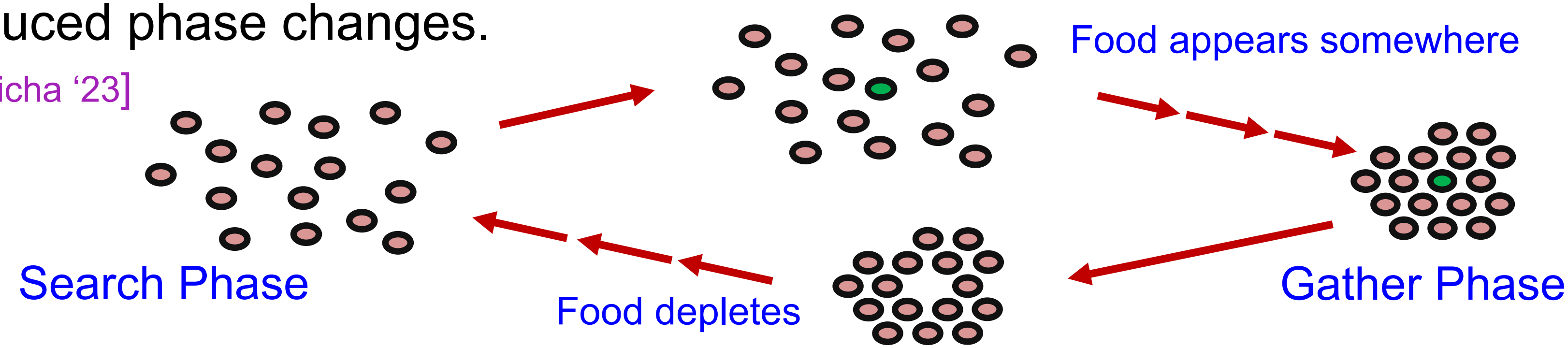
- Alignment [Kedia, Oh, R. ’22]



- Shortcut bridging [Arroyo, Cannon, Daymude, R., Richa, ’17]

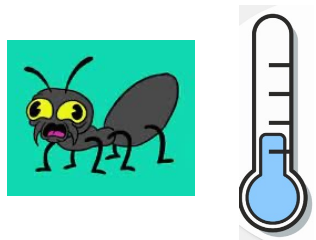


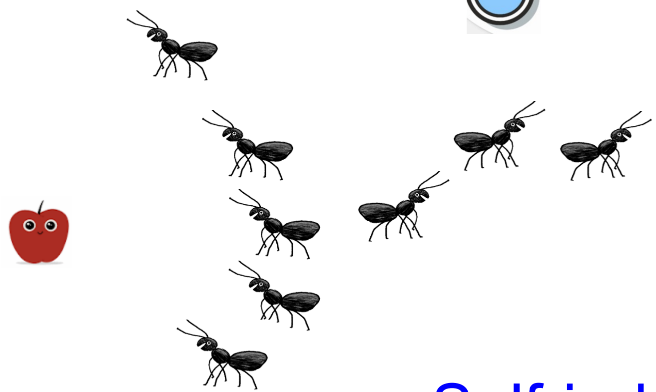
- Phototactic locomotion [Savoie, Cannon, Daymude, Warkentin, Li, Richa, R., Goldman ’18]
- Self-induced phase changes.
  [Oh, R. Richa ‘23]



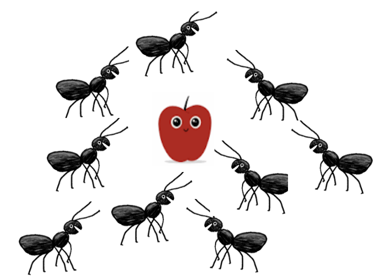
Foraging

$\lambda < 2.17$



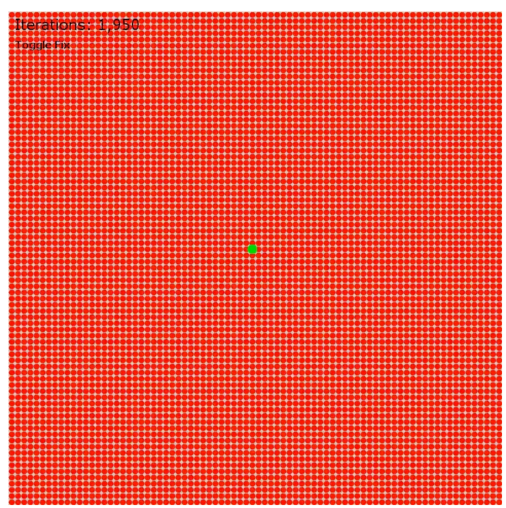


$\lambda > 3.4$



Self-induced phase changes?

Green = food
Red = compression
Yellow = dispersion
Purple = dispersion



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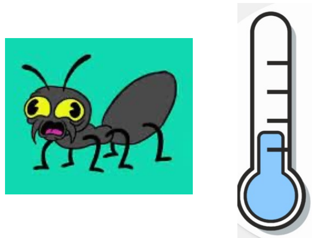

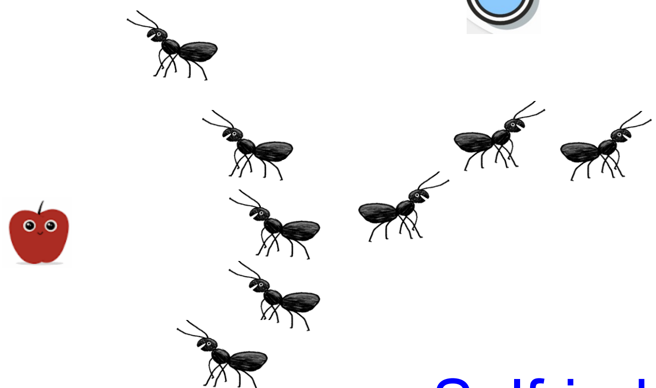

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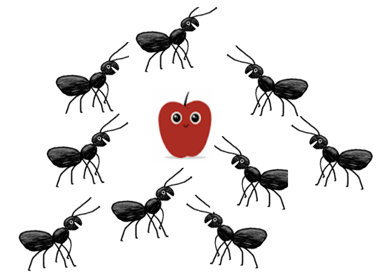
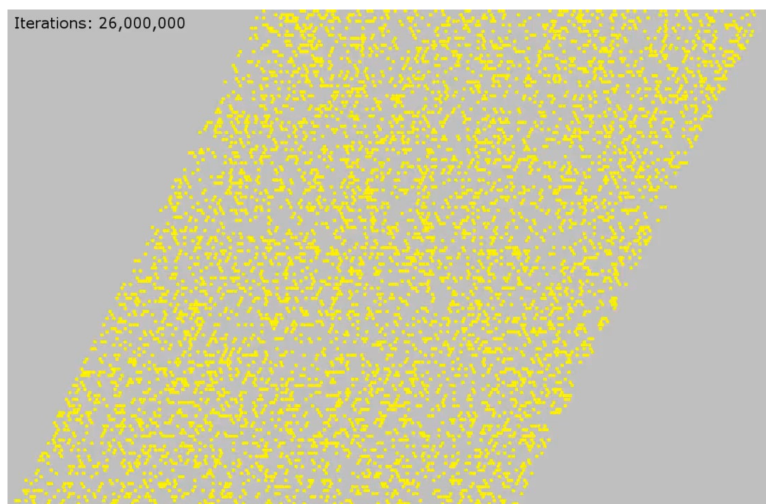


Self-induced phase changes

[Oh, R., Richa, 23]

Use careful message passing.



The real world is not reversible!

Nonreversible
Local rules

→

Global properties

~~***Boltzmann Distribution***~~

$$\Pi(\sigma) = e^{-\beta E(\sigma)}/Z$$

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HOME > SCIENCE > VOL. 371, NO. 6524 > LOW RATTLING: A PREDICTIVE PRINCIPLE FOR SELF-ORGANIZATION IN ACTIVE COLLECTIVES

REPORT

Low rattling: A predictive principle for self-organization in active collectives

PAVEL CHVYKOV, THOMAS A. BERRUETA, AKASH VARDHAN, WILLIAM SAVOIE, ALEXANDER SAMLAND, TODD D. MURPHEY, KURT WIESENFELD, DANIEL I. GOLDMAN, AND JEREMY L. ENGLAND Authors Info & Affiliations

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Real Systems of Collective Models

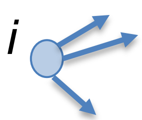
[Calvert, R. '24]

Nonreversible Local rules → ***Global properties***

New "Boltzmann-like" Distributions

(for continuous time Markov Chains)

Local part: exit rates



$$q_i = \Sigma_j \, q_{ij}$$

Nonlocal part: "jump chain"

$$p_{ij} = \frac{q_{ij}}{q_i} \; ; \quad \Psi_i$$

Stationary dist

Note: $\pi_i = \dfrac{\Psi_i}{q_i} / Z.$

The real world is not reversible!

[Calvert, R. '24]

Nonreversible
Local rules

→

Global properties

Rattling Distribution

$$\Pi(\sigma) = e^{-\gamma R(\sigma)}/Z$$

$$r^2 = \frac{\mathbf{Var}(\log \psi_I)}{\mathbf{Var}(\log q_I)} \quad \text{and} \quad \widetilde{\rho} = \mathbf{Corr}(\log \psi_I, -\log q_I).$$

The correlation ρ *of* $\log \pi_I$ *and* $-\log q_I$ *satisfies*

$$\rho = \frac{1+\widetilde{\rho} r}{\sqrt{1+2\widetilde{\rho} r + r^2}}.$$

The best linear approximation of $\log \pi_I$ *by* $-\log q_I$ *has an MSE of* $(1-\widetilde{\rho}^2)\,\mathbf{Var}(\log \psi_I)$ *and a slope of* $\gamma = 1 + \widetilde{\rho} r$.

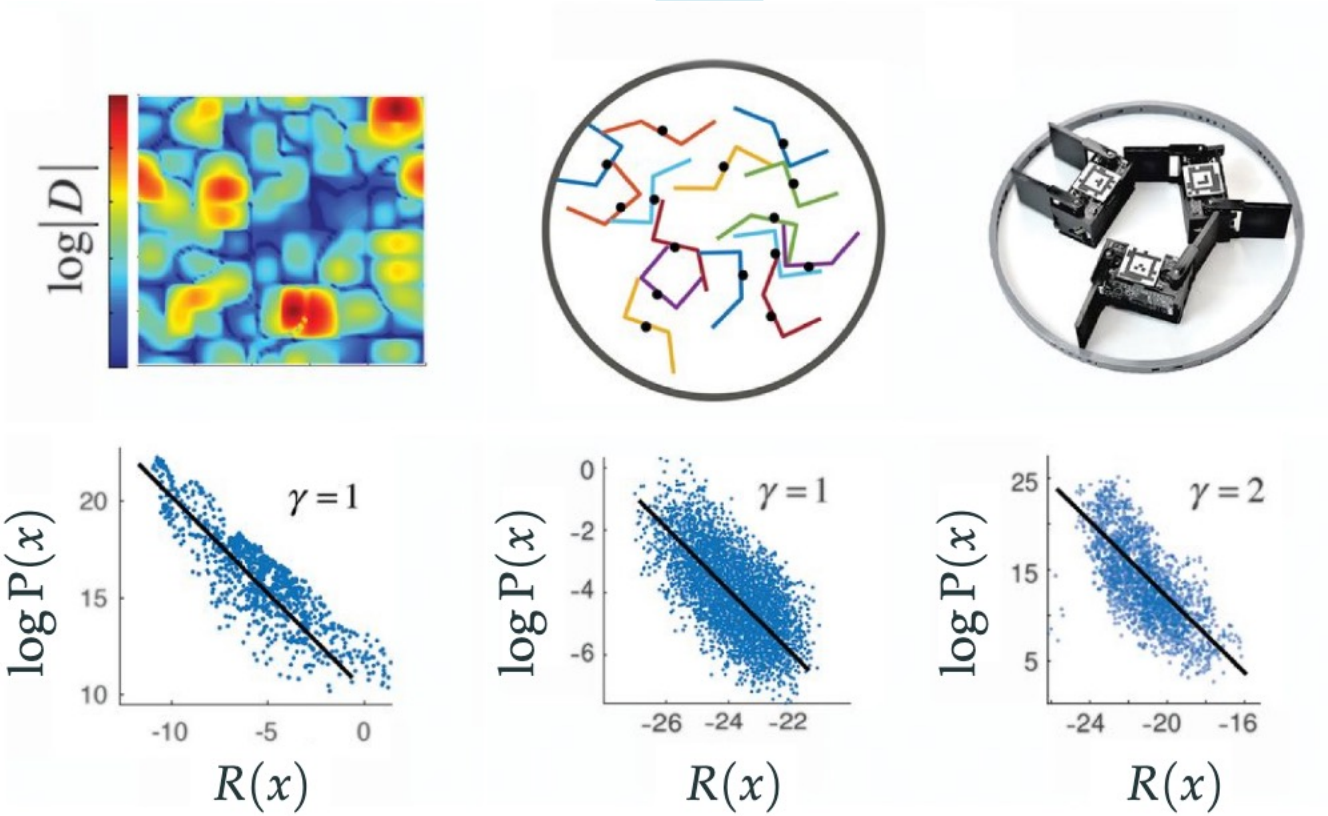
The real world is not reversible!

[Calvert, R. '24]

Nonreversible
Local rules

→

Global properties



(Adapted from Chvykov et al.)

Takeaways

- Equilibrium systems are so nice, it's magic!
- ...But many nonequilibrium systems also have rich structures connecting local features to global behaviors.
- Emergent properties of collectives can be useful design tools.
- Collectives define a rich class of "stat. phys."-type problems based on stochastic, distributed algorithms.

Thank you !

Questions?