Fall 2024

Math 553: Asymptotic Analysis and Perturbation Methods

"An easily understood, workable ~~falschood~~(estimate) is more useful than a complex, incomprehensible ~~truth~~(exact solution)."[1]

MATH 553 ASYMP/PERTURBATION METHODS [5043]
Fall 2024, WF 3:05-4:20 pm, Room 227 Physics Building
`http://www.math.duke.edu/~witelski/553`

Dr. Thomas Witelski
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Textbook: *Advanced Mathematical Methods for Scientists and Engineers* by C. M. Bender, S. A. Orszag, Springer-Verlag (1998) ("B&O")

Web resources[2]: **Canvas MATH 553.01.Fa24** `https://canvas.duke.edu/` – notes and homeworks

Background: Prerequisites include undergraduate-level background in ordinary differential differential equations (Math 353, 356 or higher) and basic background in multi-variable calculus (line integrals or contour integrals from complex variables). Material on complex variables will be concisely reviewed when needed in the course.

Course Grade: Based on weekly problem sets (70%) and the **Final Exam** (30%): **Monday, December 16, 2024, 9am-Noon**.

Lectures run to the last day of classes: **Friday Dec 6th**.

For Math Ph.D. students: Qualifying Requirement credit will generally correspond to overall **A**-level performance in the course.

Homework:[3] Assignments to be submitted using **Gradescope.com**. No unexcused late assignments will be accepted without prior approval. You are encouraged to discuss the homework problems with your classmates, but **your final written submission must be the product of your own independent work.**

Office hours: (Schedule to be announced) or by appointment(send email)

Reference books: B&O is the only required textbook for this course, supplementary notes will be made available when needed. Some other books that may be helpful for additional explanations or examples:

- *Perturbation Methods* by E. J. Hinch
- *Introduction to Perturbation Methods* by M. H. Holmes
- *Applied Asymptotic Analysis* by P. D. Miller
- *Introduction to Perturbation Techniques* by A. H. Nayfeh

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[1] Paraphrased from a quote from the Internet...

[2] Back-up webpage: `https://math.duke.edu/~witelski/553`

[3] The pledge to obey the details of the **Duke Community Standard** for conduct and academic work will be assumed in full effect throughout this course: *"I have adhered to the Duke Community Standard in completing this assignment."* If a student is found responsible through the Office of Student Conduct for academic dishonesty on a graded item in this course, the student will receive a score of zero for that assignment.

Overview

Asymptotic analysis and perturbation methods provide powerful techniques for obtaining simple analytical forms to reliably represent solutions to complicated problems in a range of different applied and mathematical settings.

This course will cover material on constructing asymptotic expansions for the solution of nonlinear algebraic equations, regular and singular perturbations problems, perturbations of matrix eigenvalue problems, asymptotics of integrals (Fourier and Laplace transforms), and solutions of differential equations (WKB theory, eigenvalue problems, multiple-scale analysis, boundary layers, and matched asymptotic expansions).

Course Outline

(1) **Introduction** (Lecture notes)

- Asymptotic approximations of functions
- Perturbation parameters, limits, and asymptotic relations: $O, o, \sim, \gg$
- Asymptotic series expansions: Taylor, Laurent, Frobenius, General Asymptotic
- Perturbation methods for solving algebraic equations
- Solution of perturbed matrix eigenvalue problems
- Regular and Singular perturbation problems

(2) **Asymptotics for integrals** B&O Chapter 6

- Elementary methods
- Watson's lemma for Laplace-type integrals
- Stationary phase for Fourier-type integrals
- Steepest descent and saddle points in the complex plane

(3) **Asymptotics for Ordinary Differential Equations I** B&O Chapters 3, 10

- Local expansions of solutions: regular and irregular singular points
- Irregular singular points at infinity
- The WKBJ method

(4) **Asymptotics for Ordinary Differential Equations II** B&O Chapters 9, 11

- Matched asymptotic expansions
- Singular perturbations: boundary layers
- Multiple scale expansions: nonlinear oscillators
  - Introduction to averaging and homogenization
  - Introduction to long-wave expansions
