Math 553: Asymptotics and Perturbation Methods   Fall 2024

Problem Set 3   **Assigned Fri Sep 13**   **Due Sat Sep 21**

Basic methods for Asymptotics of Integrals

-1. Reading : Bender and Orszag, sections 6.1–6.3 pages 247–261. (See E. J. Hinch, Perturbation Methods, section 3.4 on nonlocal integrals)

0. For all of the integrals, $I(x)$, in this homework, if the leading order term in the AE is a constant, then continue to next order to find the first non-trivial dependence on x.

1. Bender and Orszag, page 307, problem 6.7a,b,c,d,h.
   For (d), express the leading constant in terms of an exponential integral, see page 575.

2. Bender and Orszag, page 308, problem 6.17a.

3. Bender and Orszag, page 308, problem 6.18b
   Either use integration by parts, or change to an integral on $\int_x^\infty$ via the change of variables $u = xt$ and then introduce a δ breakpoint, or see page 252.

4. Consider the nearly-singular integral

$$I(\epsilon) = \int_0^1 \frac{\cos(4x)}{e^{3x} - 1 + 5\epsilon}\, dx \qquad \epsilon \to 0^+$$

   (a) In HW#1 Question 2, we were able to obtain the asymptotic expansion for an integral in terms of its Taylor series. What is wrong with writing $I(\epsilon) \sim I(0) + I'(0)\epsilon + O(\epsilon^2)$ for this problem? What is the integral for $I'(0)$?

   (b) Determine the leading order term in the expansion of $I(\epsilon)$ for $\epsilon \to 0$.