Math 553: Asymptotics and Perturbation Methods Fall 2024

Problem Set 7 **Assigned Fri Oct 18** **Due Sat Oct 26**

Asymptotics of solutions to ODE's at irregular singular points

Some notes for before you begin the problems:

-3. When expanding $S(x)$, to demonstrate that you have obtained all of the terms needed for the leading order form of $y_0(x)$, you should expand out to the first term in the regular ($O(1)$) part of S.

-2. You can use (3.4.28), but state clearly how the problem matches the Schrodinger equation and satisfies the conditions to make the result valid, or else derive the expansion for $S(x)$ starting from $y = e^S$ as usual.

-1. Be careful about whether the problems ask for the $x \to 0^+$ limit (sect 3.4) or the $x \to \infty$ limit (sect 3.5).

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0. Reading: Bender and Orszag, section 3.4, 3.5.
1. Bender and Orszag, page 140, problem 3.33 b,f.
2. Bender and Orszag, page 140, problem 3.37.
3. Bender and Orszag, page 140, problem 3.39 a,b,d,g.
4. Bender and Orszag, page 141, problem 3.46 a.
5. Bender and Orszag, page 140, problem 3.32.
6. (optional, extra credit) Show that you can obtain the leading order solution to the nonlinear equidimensional-in-y ODE:

$$y\frac{d^2y}{dx^2} = x^3\left(\frac{dy}{dx}\right)^2 \qquad x \to \infty$$

using the ansatz $y = e^{S(x)}$.

Note: This problem shows that this approach can be applied as long as you can factor out the e^S, i.e. for linear homogeneous equations and special forms of nonlinear equations. While it cannot be directly applied to solving linear inhomogeneous equations, later we will see that using techniques (variation of parameters or Green's functions) that construct the overall solution from homogeneous solutions, the $y = e^S$ solutions can be very useful more broadly.