Problem at the end of class on October 3.

Problem. Find z_x and z_y assuming $z = f(x, y)$ satisfies
$$xyz = \sin(xyz).$$

Solution. We have
$$\begin{aligned}
0 &= \frac{\partial}{\partial x}(xyz - \sin(xyz)) \\
&= yz + xyz_x - (yz + xyz_x)\cos(xyz) \\
&= (z + xz_x)y(1 - \cos(xyz))
\end{aligned}$$

and
$$\begin{aligned}
0 &= \frac{\partial}{\partial y}(xyz - \sin(xyz)) \\
&= xz + xyz_y - (xz + xyz_y)\cos(xyz) \\
&= (z + yz_x)x(1 - \cos(xyz)).
\end{aligned}$$

So, assuming
$$x \neq 0, \quad y \neq 0, \quad 1 - \cos(xyz) \neq 0,$$
(a point we will address later) we find that
$$z_x = -\frac{z}{x} \quad \text{and} \quad z_y = -\frac{z}{y}.$$