Math 103.02 Quiz Four

I have neither given nor received aid in the completion of this test.
Signature:

Let

$$\mathbf{r}(t) = \left(\frac{t^2}{2}, \sin t, t\right) \quad \text{for } t \in \mathbb{R}.$$

Compute

$$\mathbf{v}, \quad |\mathbf{v}|, \quad \mathbf{T}, \quad \mathbf{a}, \quad |\mathbf{v}|', \quad \mathbf{N}, \quad \kappa$$

at $t = 0$.

Solution. We have

$$\mathbf{v}(t) = \mathbf{r}'(t) =< t, \cos t, 1 > \quad \text{and} \quad \mathbf{a}(t) = \mathbf{v}'(t) =< 1, -\sin t, 0 > .$$

(*No more differentiation is necessary!*)

Thus

$$\begin{aligned}
\mathbf{v}(0) &=< 0, 1, 1 >, \\
|\mathbf{v}|(0) &= \sqrt{2}, \\
\mathbf{a}(0) &=< 1, 0, 0 >, \\
|\mathbf{v}|'(0) &= \mathbf{a}_T = \mathbf{a} \bullet \mathbf{T} = 0, \\
\mathbf{N}(0) &= \frac{\mathbf{a} - \mathbf{a}_T \mathbf{T}}{|\mathbf{a} - \mathbf{a}_T \mathbf{T}|}(0) =< 1, 0, 0 >, \\
\kappa(0) &= \frac{\mathbf{a} \bullet \mathbf{N}}{|\mathbf{v}|^2}(0) = \frac{1}{2}.
\end{aligned}$$