# Math 103.02 Quiz Six

**I have neither given nor received aid in the completion of this test.**
**Signature:**

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Let $T$ be the triangle with vertices $(-1,0)$, $(0,1)$ and $(1,0)$. Compute

$$\int\int_T x^2+2y^2\,dxdy.$$

**Solution.** Since

$T=\{(x,y):-1\le x\le 0 \text{ and } 0\le y\le x+1\}\cup\{(x,y):0\le x\le 1 \text{ and } 0\le y\le 1-x\}$

we have

$$\begin{aligned}\int\int_T x^2+2y^2\,dxdy &= \int_{-1}^{0}\left(\int_0^{x+1} x^2+2y^2\,dy\right)dx+\int_0^1\left(\int_0^{1-x} x^2+2y^2\,dy\right)dx\\ &= \int_{-1}^{0} x^2y+\frac{2y^3}{3}\Big|_{y=0}^{y=x+1}dx+\int_0^1 x^2y+\frac{2y^3}{2}\Big|_{y=0}^{y=1-x}dx\\ &= \int_{-1}^{0} x^2(x+1)+\frac{2(x+1)^3}{3}\,dx+\int_0^1 x^2(1-x)+\frac{2(1-x)^3}{2}\,dx\\ &= \frac{1}{4}+\frac{1}{4}\\ &= \frac{1}{2}.\end{aligned}$$

Alternatively,

$$T=\{(x,y):0\le y\le 1 \text{ and } y-1\le x\le 1-y\}$$

so

$$\begin{aligned}\int\int_T x^2+2y^2\,dxdy &= \int_0^1\left(\int_{y-1}^{1-y} x^2+2y^2\,dx\right)dy\\ &= \int_0^1\left(\frac{x^3}{3}+2xy^2\Big|_{x=y-1}^{x=1-y}\right)dy\\ &= \int_0^1\left(\frac{(1-y)^3}{3}-2(1-y)y^2\right)-\left(\frac{(y-1)^3}{3}-2(y-1)y^2\right)dx\\ &= \frac{1}{2}.\end{aligned}$$