1. The whole story for two lines in $\mathbb{R}^3$.

Suppose $\mathbf{a}, \mathbf{b}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ and $\mathbf{v}$ and $\mathbf{w}$ are nonzero. Let

$$\mathbf{r}(t) = \mathbf{a} + t\mathbf{v} \quad \text{for } t \in \mathbb{R}$$

and let

$$\mathbf{s}(t) = \mathbf{b} + t\mathbf{w} \quad \text{for } u \in \mathbb{R}.$$

Let L and M be the ranges of $\mathbf{r}$ and $\mathbf{s}$, respectively. That is,

$$L = \{\mathbf{r}(t) : t \in \mathbb{R}\} \quad \text{and} \quad M = \{\mathbf{s}(u) : u \in \mathbb{R}\}.$$

We have $L \parallel M$ if and only if $\mathbf{v} \parallel \mathbf{w}$ if and only if $\mathbf{v} \times \mathbf{w} = \mathbf{0}$. If this is so then, as I hope is clear to you, $L = M$ if and only $\mathbf{b} - \mathbf{a}$ is parallel to $\mathbf{v}$ (and, therefore, $\mathbf{w}$; this is the case if and only if

$$(\mathbf{b} - \mathbf{a}) \times \mathbf{v} = \mathbf{0} = (\mathbf{b} - \mathbf{a}) \times \mathbf{w}.$$

So suppose $\mathbf{v} \times \mathbf{w} \neq \mathbf{0}$. Let

$$\mathbf{n} = \mathbf{v} \times \mathbf{w}, \quad c = \mathbf{a} \bullet \mathbf{n}, \quad d = \mathbf{w} \bullet \mathbf{n}.$$

Let

$$P = \{\mathbf{x} \in \mathbb{R}^3 : \mathbf{x} \bullet \mathbf{n} = c\} \quad \text{and let} \quad Q = \{\mathbf{x} \in \mathbb{R}^3 : \mathbf{x} \bullet \mathbf{n} = d\}.$$

Evidently

$$L \subset P, \quad M \subset Q, \quad P \parallel Q \quad \text{and } P = Q \text{ if and only if } c = d.$$

Let find t, u, v such that

$$\mathbf{a} + t\mathbf{v} = \mathbf{b} + u\mathbf{w} + v\mathbf{n}. \tag{1}$$

It is geometrically clear that t, u, v exist and are unique. Dotting (1) with $\mathbf{n}$ we find that

$$v = \frac{c - d}{|\mathbf{n}|^2}.$$

Dotting (1) with $\mathbf{v}$ and then with $\mathbf{w}$ and transposing a little we find that

$$\begin{aligned} t|\mathbf{v}|^2 - u\mathbf{v} \bullet \mathbf{w} &= (\mathbf{b} - \mathbf{a}) \bullet \mathbf{v}, \\ -t\mathbf{v} \bullet \mathbf{w} + u|\mathbf{w}|^2 &= (\mathbf{a} - \mathbf{b}) \bullet \mathbf{w}. \end{aligned} \tag{2}$$

which is eqivalent to the matrix equation

$$\begin{bmatrix} |\mathbf{v}|^2 & -\mathbf{v} \bullet \mathbf{w} \\ -\mathbf{v} \bullet \mathbf{w} & |\mathbf{w}|^2 \end{bmatrix} \begin{bmatrix} t \\ u \end{bmatrix} = \begin{bmatrix} (\mathbf{b} - \mathbf{a}) \bullet \mathbf{v} \\ (\mathbf{a} - \mathbf{b}) \bullet \mathbf{w} \end{bmatrix} \tag{3}$$

the unique solution of which, by Cramer's Rule, is

$$t = \frac{\begin{vmatrix} (\mathbf{b} - \mathbf{a}) \bullet \mathbf{v} & -\mathbf{v} \bullet \mathbf{w} \\ (\mathbf{a} - \mathbf{b}) \bullet \mathbf{w} & |\mathbf{w}|^2 \end{vmatrix}}{\begin{vmatrix} |\mathbf{v}|^2 & -\mathbf{v} \bullet \mathbf{w} \\ -\mathbf{v} \bullet \mathbf{w} & |\mathbf{w}|^2 \end{vmatrix}}, \qquad u = \frac{\begin{vmatrix} |\mathbf{v}|^2 & (\mathbf{b} - \mathbf{a}) \bullet \mathbf{v} \\ -\mathbf{v} \bullet \mathbf{w} & (\mathbf{a} - \mathbf{b}) \bullet \mathbf{w} \end{vmatrix}}{\begin{vmatrix} |\mathbf{v}|^2 & -\mathbf{v} \bullet \mathbf{w} \\ -\mathbf{v} \bullet \mathbf{w} & |\mathbf{w}|^2 \end{vmatrix}}.$$

This gives

$$t = \frac{(\mathbf{b} - \mathbf{a}) \bullet \mathbf{comp_w v}}{|\mathbf{w}|^2 |\mathbf{v} \times \mathbf{w}|} \quad \text{and} \quad u = \frac{(\mathbf{1} - \mathbf{b}) \bullet \mathbf{comp_v w}}{|\mathbf{v}|^2 |\mathbf{v} \times \mathbf{w}|}.$$

Neat, huh? But check my calculations!